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URE 6.27: Saltwater
erface in a coastal aquifer

where R is the radius of influence, r is the radial distance of (x, y) from the pumping well, and r' is the radial distance of (x, y) from the image well. From the given data, $Q_w = 20$ L/s $= 1728$ m^3/d, $K = 28$ m/d, $H = 20$ m, $T = KH = (28)(20) = 560$ m^2/d, and $R = 600$ m. Substituting the given data into the drawdown equation yields

$$s(x, y) = \frac{1728}{2\pi(560)} \ln\left(\frac{600^2}{rr'}\right) = 0.491(12.8 - \ln rr')$$

The drawdowns 30 m from the well are given in the following table:

Location	r (m)	r' (m)	s (m)
North	30	202	2.01
South	30	202	2.01
East	30	170	2.09
West	30	230	1.94

Impermeable barriers are frequently associated with the rising side of a buried valley. This situation is quite common in the northern, once-glaciated parts of the United States. Indeed, an aquifer is often cut off in two parallel directions by buried-valley walls (AWWA, 2003b).

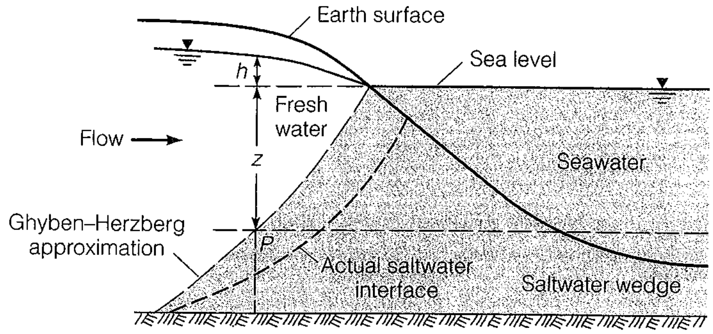
6.5.3 Other Applications

The previous examples clearly demonstrate the fundamental reasons why the method of images works, and provide sufficient guidance to apply this method to other cases. The linearity and homogeneity of the governing differential equation guarantee that superimposed solutions will also satisfy the governing differential equation. The selection of the location(s) of image wells is controlled by the requirement that the superimposed drawdowns must meet the boundary conditions. In the case of a constant-head boundary, an image well is placed to ensure zero drawdown at the constant-head boundary; in the case of an impermeable boundary, an image well is placed to ensure that the slope of the drawdown curve is zero at the impermeable boundary.

6.6 Saltwater Intrusion

In coastal aquifers, a transition region exists where the water in the aquifer changes from fresh water to saltwater. However, because saltwater is denser than fresh water, the saltwater tends to form a wedge beneath the fresh water, as shown in Figure 6.27, for the case of an unconfined aquifer. This illustration is somewhat idealized, since in reality there is not a sharp interface between fresh water and saltwater, but rather a "blurred" interface resulting from diffusion and mixing caused by the relative movement of the fresh water and saltwater. This relative movement is usually associated with tides and temporal variations in aquifer stresses. The thickness of the transition zone between fresh water and saltwater can range from a few meters to over a hundred meters (Visher and Mink, 1964). The intrusion of saltwater into coastal aquifers is generally of concern because of the associated deterioration in ground-water

E 6.27: Saltwater
ce in a coastal aquifer



quality. Since the recommended maximum contaminant level (MCL) for chloride in drinking water is 250 mg/L and a typical chloride level in seawater is 14,000 mg/L, then mixing more than 1.8% seawater with nonsaline water renders the mixture nonpotable. This percentage is even less if the fresh water contains a nonzero chloride concentration. In the United States, saltwater intrusion has resulted in the degradation of aquifers in at least 20 of the coastal states (Newport, 1977) and has been primarily caused by overpumping in sensitive portions of the aquifers. The most seriously affected states are Florida, California, Texas, New York, and Hawaii (Rail, 1989).

An approximate method for determining the location of the saltwater interface was introduced independently by Badon-Ghyben (1888) and Herzberg (1901) and is called the *Ghyben–Herzberg approximation*. Under this approximation, the pressure distribution is assumed to be hydrostatic within any vertical section of the aquifer, which implicitly assumes that the streamlines are horizontal. Under this assumption, the hydrostatic pressure at point P in Figure 6.27 can be calculated from either the fresh-water head or the saltwater head, which means that

$$\gamma_f(h + z) = \gamma_s z \tag{6.337}$$

where γ_f is the specific weight of fresh water, γ_s is the specific weight of saltwater, h is the elevation of the water table above sea level, and z is the depth of the saltwater interface below sea level. Solving Equation 6.337 for z leads to

$$\boxed{z = \frac{\gamma_f}{\gamma_s - \gamma_f}h \quad \text{or} \quad z = \frac{\rho_f}{\rho_s - \rho_f}h} \tag{6.338}$$

where ρ_f is the density of fresh water and ρ_s is the density of saltwater. This is called the *Ghyben–Herzberg equation*. Under typical conditions, $\rho_f = 1000$ kg/m^3 and $\rho_s = 1025$ kg/m^3. Substituting these values into Equation 6.338 leads to

$$z \approx 40\,h \tag{6.339}$$

which means that the saltwater interface will typically be found at a distance below sea level equal to 40 times the elevation of the water table above sea level. The Ghyben–Herzberg approximation also means that the slope of the salt water interface is 40 times greater than the slope of the water table. Near the shore, the depth to the interface predicted by the Ghyben–Herzberg approximation tends to be less than the actual depth observed in the field (Fitts, 2002). In fact, at the shoreline the

Ghyben–Herzberg approximation predicts that the saltwater interface is at sea level, while there must necessarily be a nonzero thickness of fresh water there.

Assuming that the flow in the fresh-water portion of the aquifer is horizontal and towards the coast, neglecting direct surface recharge (such as from rainfall), and assuming that there is no flow within the saltwater wedge, the flowrate, Q, of fresh water toward the coast can be estimated using the Darcy equation

$$Q = K(h + z)\frac{dh}{dx} \tag{6.340}$$

where K is the hydraulic conductivity of the aquifer, and x is the distance inland from the shoreline. Equation 6.340 uses the Dupuit approximation, which assumes horizontal flow and equates the horizontal piezometric head gradient to the slope of the water table. Combining Equations 6.340 and 6.338 yields

$$Q = K\left(\frac{\gamma_s}{\gamma_s - \gamma_f}\right)h\frac{dh}{dx} \tag{6.341}$$

and integrating Equation 6.341 yields

$$Qx = \frac{K}{2}\left(\frac{\gamma_s}{\gamma_s - \gamma_f}\right)h^2 + C \tag{6.342}$$

where C is an integration constant. Applying the boundary condition that $h = 0$ at $x = 0$ yields $C = 0$, and applying the boundary condition that $h = h_L$ at $x = L$ yields

$$\boxed{Q = \frac{K}{2L}\left(\frac{\gamma_s}{\gamma_s - \gamma_f}\right)h_L^2} \tag{6.343}$$

This equation is particularly useful in estimating the flow of fresh water toward the coast, based on the elevation, h_L, of the water at a distance L from the coast. The water-table profile can be estimated by combining Equations 6.342 (with $C = 0$) and 6.343 to yield

$$\boxed{h = h_L\sqrt{\frac{x}{L}}} \tag{6.344}$$

The results presented here demonstrate that a small number of piezometric head measurements can be used to obtain an estimate of the fresh-water discharge of an aquifer and the location of the interface between fresh water and saltwater.

EXAMPLE 6.30

Measurements in a coastal aquifer indicate that the saltwater interface intercepts the bottom of the aquifer approximately 2 km from the shoreline. If the hydraulic conductivity of the aquifer is 50 m/d and the bottom of the aquifer is 60 m below sea level, estimate the fresh-water discharge per kilometer of shoreline.

Solution From the given data: $K = 50$ m/d, $L = 2$ km $= 2000$ m, and $z = 60$ m. Assuming $\rho_f = 1000$ kg/m^3 and $\rho_s = 1025$ kg/m^3, then Equation 6.338 gives

$$z = \frac{\rho_f}{\rho_s - \rho_f} h_L$$

$$60 = \frac{1000}{1025 - 1000} h_L$$

which yields $h_L = 1.5$ m. Substituting given data into Equation 6.343 gives

$$\begin{aligned} Q &= \frac{K}{2L}\left(\frac{\gamma_s}{\gamma_s - \gamma_f}\right) h_L^2 \\ &= \frac{50}{2(2000)}\left(\frac{1025}{1025 - 1000}\right)(1.5)^2 \\ &= 1.15 \text{ m}^2/\text{d} \end{aligned}$$

Therefore, the fresh-water discharge per kilometer of shoreline is $1.15 \times 1000 = 1150$ (m^3/d)/km.

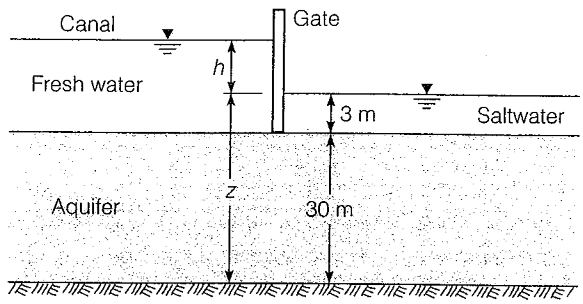
In applying the Ghyben–Herzberg approximation, Equation 6.338, it is useful to note that the assumption of horizontal flow produces acceptable results, except near the coastline where vertical flow components become significant, in which case the actual saltwater interface is expected to be found below the location predicted by the Ghyben–Herzberg equation (Bear, 1979). In the case of confined aquifers, the Ghyben–Herzberg approximation is also applicable, with the elevation of the water table replaced by the elevation of the piezometric surface. Bear and Dagan (1962) have shown that the length of saltwater intrusion into a horizontal confined aquifer of thickness b is predicted to within 5% by the Ghyben–Herzberg equation, provided that $\pi(\Delta\gamma/\gamma_f)Kb/Q > 8$, where Q is the rate of flow of fresh water per unit breadth of the aquifer, and $\Delta\gamma = \gamma_s - \gamma_f$.

Besides saltwater intrusion caused by the density difference between saltwater and fresh water, a second important mechanism for saltwater intrusion is associated with the construction of unregulated coastal drainage canals. These canals allow the inland penetration of saltwater via tidal inflow and subsequent leakage of saltwater from the canals into the aquifer. To prevent saltwater intrusion in coastal drainage canals, salinity-control gates are typically placed at the downstream end of the canal to maintain a fresh-water head (on the upstream side of the gate) over the sea elevation (on the downstream side of the gate). The fresh-water head should be sufficient to prevent saltwater intrusion in accordance with the Ghyben–Herzberg equation. During periods of high runoff and when the stages in the canals are above a prescribed level, then the canal gates are opened to permit drainage while maintaining a fresh-water head that is sufficient to prevent saltwater intrusion.

EXAMPLE 6.31

Consider the gated canal in a coastal aquifer illustrated in Figure 6.28. If the aquifer thickness below the canal is 30 m, and at high tide the depth of seawater on the

FIGURE 6.28: Gated canal



downstream side of the gate is 3 m, find the depth of fresh water on the upstream side of the gate that must be maintained to prevent saltwater intrusion.

Solution The elevation of the fresh-water surface at the upstream side of the gate must be sufficient to maintain the saltwater interface at a depth of 33 m below sea level. According to the Ghyben–Herzberg equation (Equation 6.338), the height of the fresh-water surface above sea level, h, is given by

$$h = \frac{\rho_s - \rho_f}{\rho_f} z$$

where ρ_s and ρ_f are the densities of saltwater and fresh water, respectively, and z is the depth of the interface below sea level. Substituting $\rho_s = 1025$ kg/m^3, $\rho_f = 1000$ kg/m^3, and $z = 33$ m yields

$$h = \frac{1025 - 1000}{1000} 33$$
$$= 0.83 \text{ m}$$

Therefore, the fresh water on the upstream side of the gate must be held at 0.83 m above the sea level on the downstream side of the gate. The total depth of fresh water in the canal is 3 m + 0.83 m = 3.83 m.

In addition to salinity-control gates in coastal drainage channels, other methods of controlling saltwater intrusion include modification of pumping patterns, creation of fresh-water recharge areas, and installation of extraction and injection barriers. Extraction barriers are created by maintaining a continuous pumping trough with a line of wells adjacent to the sea, and injection barriers are created by injecting high-quality fresh water into a line of recharge wells to create a high-pressure ridge. In extraction barriers, seawater flows inland toward the extraction wells and fresh water flows seaward toward the extraction wells. The pumped water is brackish and is normally discharged to the sea.

Whenever water-supply wells are installed above the saltwater interface, the pumping rate from the wells must be controlled so as not to pull the saltwater up into the well. The process by which the saltwater interface rises in response to pumping is called *upconing*. This phenomenon is illustrated in Figure 6.29. Schmorak and Mercado (1969) proposed the following approximation of the rise height, z, of the

FIGURE 6.29: Upconir
partially penetrating

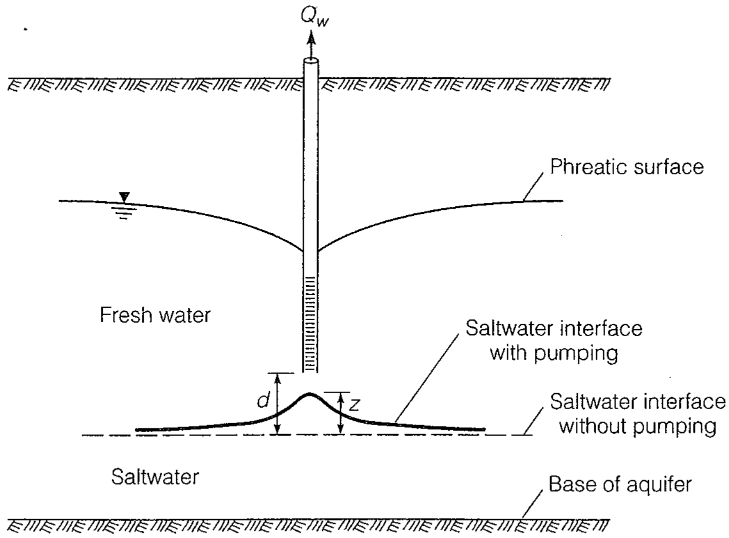


FIGURE 6.29: Upconing under a partially penetrating well

saltwater interface in response to pumping:

$$z = \frac{Q_w}{2\pi d K_x(\Delta\rho/\rho_f)} \tag{6.345}$$

where Q_w is the pumping rate, d is the depth of the saltwater interface below the well before pumping, K_x is the horizontal hydraulic conductivity of the aquifer, ρ_f is the density of fresh water, and $\Delta\rho$ is defined by

$$\Delta\rho = \rho_s - \rho_f \tag{6.346}$$

where ρ_s is the saltwater density. Equation 6.345 incorporates both the Dupuit and Ghyben–Herzberg approximations, and therefore care should be taken in cases where significant deviations from these approximations occur. Experiments have shown that whenever the rise height, z, exceeds a critical value, then the saltwater interface accelerates upward toward the well. This critical rise height has been estimated to be in the range $0.3d$ to $0.5d$ (Todd, 1980). Taking the maximum allowable rise height to be $0.3d$ in Equation 6.345 corresponds to a pumping rate, Q_{max}, given by

$$Q_{\text{max}} = 0.6\pi d^2 K_x \frac{\Delta\rho}{\rho_f} \tag{6.347}$$

Therefore, as long as the pumping rate is less than or equal to Q_{max}, pumping of fresh water above a saltwater interface remains viable, although pumping rates must remain steady to avoid blurring the interface. For anisotropic aquifers in which the vertical component of the hydraulic conductivity is less than the horizontal component, a maximum well discharge larger than that given by Equation 6.347 is possible (Chandler and McWhorter, 1975).

EXAMPLE 6.32

A well pumps at 5 L/s in a 30-m thick coastal aquifer that has a hydraulic conductivity of 100 m/d. How close can the saltwater wedge approach the well before the quality of the pumped water is affected?

Solution From the given data: $Q_w = 5$ L/s $= 432$ m^3/d, $K_x = 100$ m/d, $\rho_f =$ 1000 kg/m^3, $\rho_s = 1025$ kg/m^3, and $\Delta\rho = \rho_s - \rho_f = 1025 - 1000 = 25$ kg/m^3. Equation 6.347 gives the minimum allowable distance of the saltwater wedge from the well as

$$d = \sqrt{\frac{Q_{\max}}{0.6\pi K_x \, \Delta\rho/\rho_f}}$$

If $Q_{\max} = 432$ m^3/d, then

$$d = \sqrt{\frac{432}{0.6\pi(100)(25)/1000}} = 9.6 \text{ m}$$

Therefore, the quality of pumped water will be impacted when the saltwater interface is located 9.6 m below the pumping well.

In cases where the pumping well fully penetrates the aquifer, the drawdown induced by the well must be limited to ensure that the toe of the saltwater wedge does not intersect the well (Mantoglou, 2003). In this case, the Ghyben–Herzberg equation can be used to estimate the limiting drawdown.

Saline ground water is a general term used to describe ground water containing more than 1000 mg/L of total dissolved solids. There are several classification schemes for ground water based on total dissolved solids, and a widely cited one, initially proposed by Carroll (1962), is given in Table 6.14. Intruded seawater has a total dissolved solids concentration of 35,000 mg/L and is classified as saline ground water. Other forms of saline ground water include *connate water** that was originally buried along with the aquifer material, water salinized by contact with soluble salts in the porous formation where it is situated, and water in regions with shallow water tables where evapotranspiration concentrates the salts in solution.

6.7 Ground-Water Flow in the Unsaturated Zone

Porous media in which the void spaces are not completely filled with water are called *unsaturated*, and an *unsaturated zone* is typically found between the ground surface and

TABLE 6.14: Classification of Saline Ground Water

Classification	Total dissolved solids (mg/L)
Fresh water	0–1000
Brackish water	1000–10,000
Saline water	10,000–100,000
Brine	> 100,000

Source: Carroll (1962).

*The word *connate* is derived from the latin word *connatus*, which means "born together."

FIGURE 6.30: Capillary ri

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