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7.4 EMPIRICAL RELATIVE FREQUENCY RELATIONS

A series of N observations may be ranked in descending order with the highest value assigned a rank m of 1 and the smallest assigned a rank m of N. The probability P_m that the observation with rank m is equaled or exceeded becomes

$$P_m = \left(\frac{m}{N}\right)_{N \to \infty} \tag{7.8}$$

as the number of observations (sample size) N approaches infinity. Without N approaching infinity, the relative frequency relation

$$P_m = \frac{m}{N} \tag{7.9}$$

provides an estimate of the probability of observation m being equaled or exceeded, with the accuracy improving with increasing sample size. Equation 7.9 will assign an exceedance probability of 1.0 to the smallest of the N observations, indicating a zero probability of obtaining a value less than those observed, which is usually not correct. Other frequency relations (Eqs. 7.10 and 7.11) have been formulated that eliminate assigning an exceedance probability of 1.0 to an observation. Empirical frequency relations are often called plotting position formulas because they are used to plot observations on probability graph paper.

The general form of most plotting position formulas is as follows:

$$P_m = \frac{m - a}{N + b} \tag{7.10}$$

Equation 7.9 with $a = b = 0$ and the Weibull formula (Eq. 7.11) with $a = 0$ and $b = 1$ are the most commonly used forms of Eq. 7.10.

$$P_m = \frac{m}{N + 1} \tag{7.11}$$

The Weibull formula may be expressed in terms of either annual exceedance probability or recurrence interval T for rank m and number of years of observation N.

$$P = \frac{m}{N + 1} \quad \text{and} \quad T = \frac{N + 1}{m} \tag{7.12}$$

The exceedance probability may be expressed as an exceedance frequency in percent by multiplying P_m and P from Eqs. 7.8–7.12 by 100 percent.

SOURCE: Wurbs & James, 2002

TABLE 3.4 Plotting Position Formulas

Method	Solve for $P(X > x)$	For $m = 1$ and $n = 10$ P	T
California	$\frac{m}{n}$	.10	10
Hazen	$\frac{2m - 1}{2n}$	.05	20
Beard	$1 - (0.5)^{1/n}$	.067	14.9
Weibull	$\frac{m}{n + 1}$	.091	11
Chegadayev	$\frac{m - 0.3}{n + 0.4}$	.067	14.9
Blom	$\frac{m - \frac{3}{8}}{n + \frac{1}{4}}$	.061	16.4
Tukey	$\frac{3m - 1}{3n + 1}$	.065	15.5

where n = the number of years of record
m = the rank
a = a parameter depending on n as follows:

n	10	20	30	40	50
a	0.448	0.443	0.442	0.441	0.440
n	60	70	80	90	100
a	0.440	0.440	0.440	0.439	0.439

SOURCE: Viessman, W., Jr. and G. L. Lewis
Introduction to Hydrology
Prentice-Hall, 2003

EMPIRICAL METHOD

TABLE 7.1 MAXIMUM ANNUAL DISCHARGE IN THE MISSISSIPPI RIVER AT ST. LOUIS

Year	Flow (m^3/s)	Year	Flow (m^3/s)	Year	Flow (m^3/s)
1933	12,400	1955	8,800	1977	11,000
1934	6,260	1956	5,860	1978	16,200
1935	18,500	1957	9,620	1979	19,500
1936	9,450	1958	14,300	1980	9,930
1937	10,600	1959	10,300	1981	14,400
1938	12,300	1960	19,000	1982	20,700
1939	15,100	1961	16,700	1983	20,300
1940	5,240	1962	16,700	1984	16,400
1941	14,000	1963	8,510	1985	19,500
1942	18,900	1964	8,710	1986	20,500
1943	23,700	1965	15,600	1987	11,900
1944	23,700	1966	10,500	1988	8,850
1945	17,400	1967	15,000	1989	9,280
1946	14,200	1968	9,790	1990	17,000
1947	22,300	1969	17,500	1991	12,400
1948	17,900	1970	15,300	1992	14,600
1949	12,000	1971	11,900	1993	30,600
1950	13,100	1972	11,500	1994	17,000
1951	22,200	1973	24,200	1995	22,500
1952	19,400	1974	16,500	1996	17,400
1953	10,400	1975	13,700	1997	15,400
1954	8,230	1976	12,700	1998	15,500

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Example 7.4

The Weibull formula is used to develop a frequency relationship for peak flows on the Mississippi River at St. Louis. The observations of peak annual flows in Table 7.1 are rearranged in ranked order in Table 7.2. Annual exceedance probabilities for each observed flow are assigned using the Weibull formula. The flows are plotted with their assigned exceedance frequencies on normal probability paper in Fig. 7.1 and on log-normal probability paper in Fig. 7.2. These plots and the other information included in Figs. 7.1 and 7.2 are discussed in Sections 7.6 and 7.7.

TABLE 7.2 FLOWS FROM TABLE 7.1 IN RANKED ORDER WITH P AND T FROM WEIBULL FORMULA (EXAMPLE 7.4)

Rank m	$P = m/67$	$T = 67/m$	Flow (m^3/s)	Year	Rank m	$P = m/67$	$T = 67/m$	Flow (m^3/s)	Year
1	0.0149	67.0	30,600	1993	34	0.508	1.97	14,600	1992
2	0.0299	33.5	24,200	1973	35	0.522	1.91	14,400	1981
3	0.0448	22.3	23,700	1943	36	0.537	1.86	14,300	1958
4	0.0597	16.8	23,700	1944	37	0.552	1.81	14,200	1946
5	0.0746	13.4	22,500	1995	38	0.567	1.76	14,000	1941
6	0.0896	11.2	22,200	1951	39	0.582	1.72	13,700	1975
7	0.1045	9.6	20,700	1982	40	0.597	1.68	13,100	1950
8	0.119	8.4	20,500	1986	41	0.612	1.63	12,700	1976
9	0.134	7.4	20,300	1947	42	0.627	1.60	12,400	1933
10	0.149	6.7	20,300	1983	43	0.642	1.56	12,400	1991
11	0.164	6.1	19,500	1979	44	0.657	1.52	12,300	1938
12	0.179	5.6	19,500	1985	45	0.672	1.49	12,000	1949
13	0.194	5.2	19,400	1952	46	0.687	1.46	11,900	1971
14	0.209	4.8	19,400	1960	47	0.702	1.43	11,900	1987
15	0.224	4.5	18,900	1942	48	0.716	1.40	11,500	1972
16	0.239	4.2	18,500	1935	49	0.731	1.37	11,000	1977
17	0.254	3.9	17,900	1948	50	0.746	1.34	10,600	1937
18	0.269	3.7	17,500	1969	51	0.761	1.31	10,500	1966
19	0.284	3.5	17,400	1945	52	0.776	1.29	10,400	1953
20	0.299	3.4	17,400	1996	53	0.791	1.26	10,300	1959
21	0.313	3.2	17,000	1990	54	0.806	1.24	9,930	1980
22	0.328	3.0	17,000	1994	55	0.821	1.22	9,790	1968
23	0.343	2.9	16,700	1961	56	0.836	1.20	9,620	1957
24	0.358	2.8	16,700	1962	57	0.851	1.18	9,450	1936
25	0.373	2.7	16,500	1974	58	0.866	1.16	9,280	1989
26	0.388	2.6	16,400	1984	59	0.881	1.14	8,850	1988
27	0.403	2.5	16,200	1978	60	0.896	1.12	8,800	1955
28	0.418	2.4	15,600	1965	61	0.910	1.10	8,710	1964
29	0.433	2.3	15,500	1998	62	0.925	1.08	8,510	1963
30	0.448	2.2	15,400	1997	63	0.940	1.06	8,230	1954
31	0.463	2.2	15,300	1970	64	0.955	1.05	6,260	1934
32	0.478	2.1	15,100	1939	65	0.970	1.03	5,860	1956
33	0.493	2.0	15,000	1967	66	0.985	1.02	5,240	1940

With 66 years of observations, the recurrence interval assigned to the highest observed discharge is

$$T = \frac{N+1}{m} = \frac{66+1}{1} = 67 \text{ years}$$

with an associated exceedance probability of

$$P = \frac{m}{N+1} = \frac{1}{66+1} = 0.0149$$

SOURCE: Wurbs & James, 2002

7.6 FREQUENCY GRAPHS

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The frequency analysis for the Mississippi River at St. Louis is presented graphically in Figs. 7.1 and 7.2. These graphs were printed from the Hydrologic Engineering Center-Flood Frequency Analysis (HEC-FFA) (Hydrologic Engineering Center, 1992) computer program discussed in Section 7.7.1. The confidence limits on the frequency curves are discussed in Section 7.7.2. The Weibull plotting positions from Section 7.4 and analytical flow frequency curves from Section 7.5 are discussed in the following paragraphs.

In Fig. 7.1, flows are on an arithmetic scale versus exceedance frequency on a normal probability scale. In the log-normal graph of Fig. 7.2, the flows are on a logarithmic scale. The Weibull plotting positions from Table 7.2 are plotted on both graphs as discussed in Section 7.4. A curve could be drawn through the 66 data points manually based on judgment regarding the best fit. Different people might draw the line somewhat differently. However, the frequency curve lines actually included on the two graphs are based on the analytical probability functions discussed in Section 7.5, not Weibull plotting positions. The frequency curves are fixed precisely by the analytical distributions with parameters computed from the data. The normal distribution and log-Pearson type III distribution are graphed in Figs. 7.1 and 7.2, respectively.

The normal distribution is a straight line on graph paper with an arithmetic scale versus normal probability scale. Thus, the frequency curve in Fig. 7.1 is a straight line through the 10-year and 100-year recurrence interval flows of 21,500 and 27,000 m^3/s determined in Example 7.5 or any other two points computed based on the normal probability distribution. The log-normal distribution is linear on log-normal graph paper, which has a logarithmic scale versus normal probability scale as illustrated by Fig. 7.2. Equivalently, a graph of logarithms of flows plotted on an arithmetic scale versus exceedance frequencies determined from the log-normal distribution plotted on a normal scale is linear. Although the log-normal distribution is not plotted in Fig. 7.2, it easily could be. The 10-year and 100-year flows determined in Example 7.5 define a straight line representing the log-normal distribution on log-normal graph paper.

The log-Pearson type III flow frequency curve is shown in Fig. 7.2, along with confidence limits that are discussed later in Section 7.7.2. The graph has logarithmic versus normal probability scales. With a nonzero skew coefficient, the log-Pearson type III distribution is a nonlinear curve. If the skew coefficient is zero, the log-Pearson type III distribution is equivalent to the log-normal distribution and plots as a straight line on log-normal probability paper.

The 1993 flood discussed in Section 2.2.3.2 resulted in a peak discharge of 30,600 m^3/s on August 1, 1993 at the gage on the Mississippi River at St. Louis. The log-Pearson type III curve in Fig. 7.2 indicates that 30,600 m^3/s has an exceedance frequency of about 0.4 percent ($P = 0.004$ and $T = 250$ years). This analysis addresses only peak discharge at this particular gaging station. As discussed in Section 2.2.3.2, the 1993 flood in the Midwest encompassed the Missouri and Mississippi Rivers and their tributaries in several states. Different recurrence intervals are assigned at different locations for the same flood.

SOURCE: Wurbs & James, 2002

EXAMPLES OF FREQUENCY GRAPHS
(HEC-FFA computer program)
[Hydrologic Eng. Center - Flood Frequency Analysis]

arithmetic scale

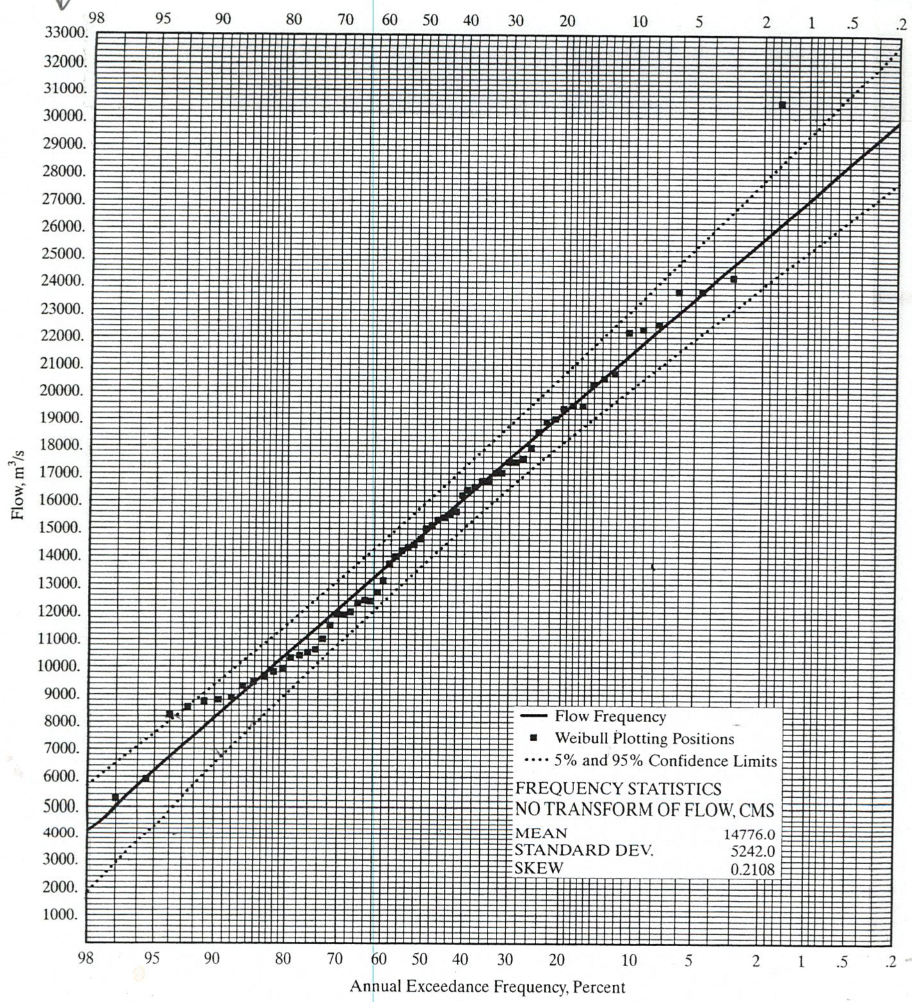

Figure 7.1 The normal frequency curve and Weibull plotting positions for peak annual flows in the Mississippi River at St. Louis are graphed on normal probability paper.

logarithmic scale

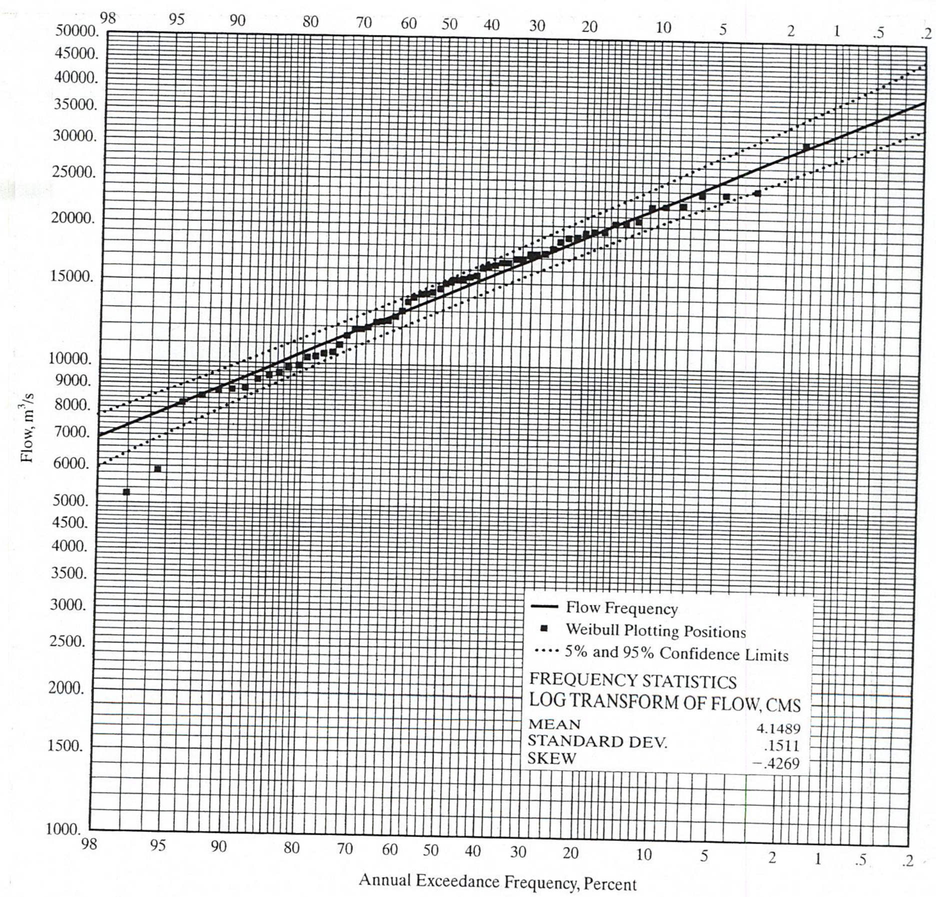

Figure 7.2 The log-Pearson type III frequency curve and Weibull plotting positions for peak annual flows in the Mississippi River at St. Louis are graphed on log-normal probability paper.