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**Table 10.1.1** Various Parameters and Statistics Used to Describe Populations and Samples

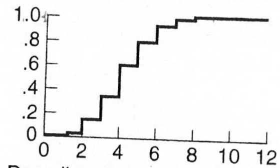
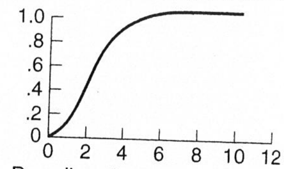
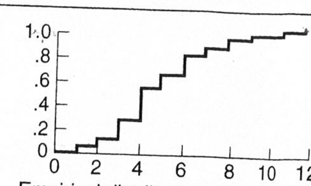
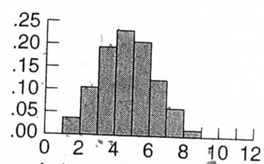
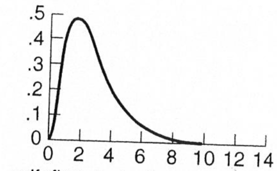
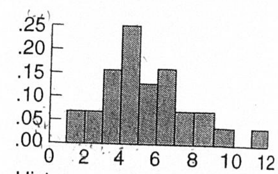
| Concept | Population Value, Discrete Case | Population Value, Continuous Case | Sample Value |
|---|---|---|---|
| Cumulative distribution function (cdf) |   Describes the probability that a random variable is less than or equal to a specified value $x$ |   Describes the probability that a random variable is less than or equal to a specified value $x$ |   Empirical distribution function (edf): describes the observed frequency of a random variable being less than or equal to a specified value $x$ |
| Probability mass function (pmf) and probability density function (pdf) |   pmf: the probability that $X$ is equal to $k$ |   pdf: first derivative of the cumulative distribution function |   Histogram: observed frequency with which random variable $X$ falls into the assigned ranges |
| Mean, average, or expected value | $\sqrt{0.368}$ | $f(x) \equiv \frac{dF(x)}{dx}$ <br> $\mu \equiv \int_{-\infty}^{\infty} x f(x)dx$ | $\bar{X} \equiv \sum_{i=1}^{n} \frac{X_i}{n}$ |
| Variance | $\sigma^2 \equiv \sum_{i=1}^{\infty} P(X = x_i)(x_i - \mu)^2$ | $\sigma^2 \equiv \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ | $S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n-1}$ |
| $k$th central moment | $M_k = \sum_{i=1}^{\infty} P(X = x_i)(x_i - \mu)^k$ | $M_k \equiv \int_{-\infty}^{\infty} (x-\mu)^k f(x)dx$ | $\tilde{M}_k \equiv \sum_{i=1}^{n} \frac{(X_i - \bar{X})^k}{n}$ |
| Standard deviation | $\sigma \equiv \sqrt{\sigma^2}$ | | $S \equiv \sqrt{S^2}$ |
| Coefficient of variation or relative standard deviation (if $\mu \neq 0$) | $CV \equiv \frac{\sigma}{\mu}$ | | $CV \equiv \frac{S}{\bar{X}}$ |
| Coefficient of skew (a measure of asymmetry) | $\gamma \equiv \frac{M_3}{\sigma^3}$ | | $G \equiv \frac{\tilde{M}_3}{S^3}$ |
| Quantiles | $x_p$ is any value of $X$ that has the properties that $P[X < x_p] \leq p$, $P[X > x_p] \leq 1 - p$ | | $\hat{X}_p$ is the $p$th quantile of edf |
| Median (useful for describing central tendency regardless of skewness) | $x_{0.5}$ Any value of X that has the property that $P[X < x_p] \leq 0.5$, $P[X > x_p] \leq 0.5$ | | $\hat{X}_{0.5}$ The middle observation in a sorted sample, or the average of the two middle observations if the sample size is even. |

**Table 10.1.1** (*Continued*)

| Concept | Population value, discrete case | Population value, continuous case | Sample value |
|---|---|---|---|
| Upper quartile, lower quartile, and hinges | Upper quartile $= x_{0.75}$ Lower quartile $= x_{0.25}$ | | Upper hinge $= \widehat{X}_{0.75}$ This is an approximation to the sample upper quartile; it is defined as the median of all sample values of $X \leq x_{0.50}$. The lower hinge, $\widehat{X}_{0.25}$>, is defined analogously. |
| Interquartile range (useful for describing spread of data regardless of symmetry) | $x_{0.75} - x_{0.25}$ Width of central region of population containing probability of 0.5 | | $\widehat{X}_{0.75} - \widehat{X}_{0.25}$ Width of central region of data set encompassing approximately half the data |

*Source:* Hirsh, et al. (1993).

**Table 10.1.2** Probability Distributions Commonly Used

| Distribution | Probability density function | Range | Parameter-Moment relations |
|---|---|---|---|
| Normal | $f(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$ | $-\infty < x < \infty$ | |
| Log-normal | $f(x) = \dfrac{1}{\sqrt{2\pi} x \sigma_{\ln x}} e^{-(\ln x - \mu_{\ln x})^2/(2\sigma^2_{\ln x})}$ | $x > 0$ | $\mu_{\ln x} = \dfrac{1}{2}\ln\left[\dfrac{\mu_x^2}{1+\Omega_x^2}\right]$ <br> $\sigma^2_{\ln x} = \ln(1+\Omega_x^2)$ <br> $\Omega_x = \sigma_x/\mu_s$ |
| Exponential | $f(x) = \lambda e^{-\lambda x}$ | $x \geq 0$ | $\lambda = \dfrac{1}{\mu_x}$ |
| Gamma | $f(x) = \dfrac{\lambda^\beta x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)}$ <br> where $\Gamma$ = gamma function | $x \geq 0$ | $\lambda = \dfrac{\mu_x}{\sigma_x^2}$, $\beta = \dfrac{\mu_x^2}{\sigma_x^2} = \dfrac{1}{C_v^2}$ |
| Extreme Value Type I (or Gumbel Distribution) | $f(x) = \dfrac{1}{\alpha} e^{-(x-\beta)/\alpha - e^{-(x-\beta)/\alpha}}$ | $-\infty < x < \infty$ | $\alpha = \sqrt{6}\sigma_x/\pi$ <br> $\beta = \mu_x - 0.5772\alpha$ |
| Log-Pearson Type III | $f(x) = \dfrac{\lambda^\beta (y-\epsilon)^{x\beta-1} e^{-\lambda(v-\epsilon)}}{x\Gamma(\beta)}$ <br> where $y = \log x$ | $\log x \geq \epsilon$ | $\lambda = \dfrac{s_y}{\sqrt{\beta}}$, <br> $\beta = \left[\dfrac{2}{G_s(y)}\right]^2$ <br> $\epsilon = \bar{y} - s_y\sqrt{\beta}$ <br> (assuming $G_s(y)$ is positive) |

## 10.3 HYDROLOGIC DESIGN FOR WATER EXCESS MANAGEMENT

*Hydrologic design* is the process of assessing the impact of hydrologic events of a water resource system and choosing values for the key variables of the system so that it will perform adequately (Chow et al., 1988). This section focuses on water excess management; however, many of the concepts are applicable to water supply (use) management.

### 10.3.1 Hydrologic Design Scale

The *hydrologic design scale* is the range in magnitude of the design variable (such as the design discharge) within which a value must be selected to determine the inflow to the system (see Figure 10.3.1). The most important factors in selecting the design value are cost and safety. The optimal magnitude for design is one that balances the conflicting considerations of cost and safety. The practical upper limit of the hydrologic design scale is not infinite, since the global hydrologic cycle is a closed system; that is, the total quantity of water on earth is essentially constant. Although the true upper limit is unknown, for practical purposes an estimated upper limit may be determined. This *estimated limiting value* (ELV) is defined as the largest magnitude possible for a hydrologic event at a given location, based on the best available hydrologic information.

The concept of an estimated limiting value is implicit in the *probable maximum precipitation* (PMP) and the corresponding *probable maximum flood* (PMF). The probable maximum precipitation is defined by the World Meteorological Organization (1983) as a "quantity of precipitation that is close to the physical upper limit for a given duration over a particular basin." Based on

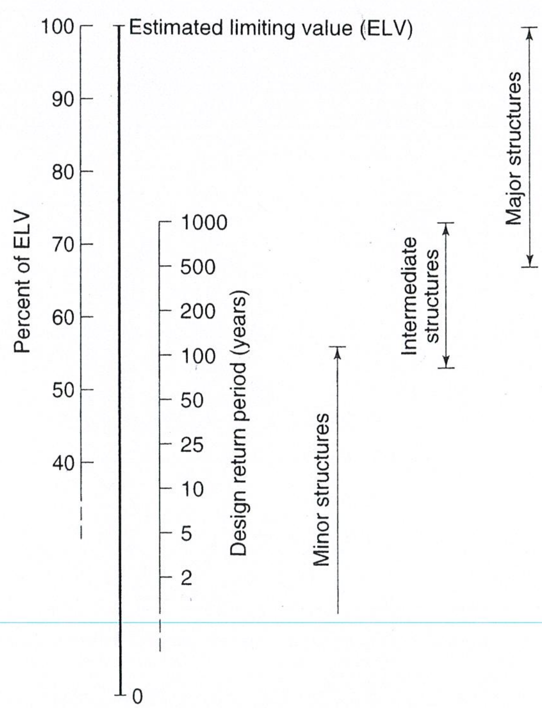



**Figure 10.3.1** Hydrologic design scale. Approximate ranges of the design level for different types of structures are shown. Design may be based on a percentage of the ELV or on a design return period. The values for the two scales shown in the diagram are illustrative only and do not correspond directly with one another (from Chow et al. (1988)).

worldwide records, the PMP can have a return period of as long as 500,000,000 years. However, the return period varies geographically. Some arbitrarily assign a return period, say 10,000 years, to the PMP or PMF, but this has no physical basis.

Generalized design criteria for water-control structures have been developed, as summarized in Table 10.3.1. According to the potential consequence of failure, structures are classified as *major, intermediate,* and *minor*; the corresponding approximate ranges on the design scale are shown in Figure 10.3.1. The criteria for dams in Table 10.3.1 pertain to the design of spillway capacities, and are taken from the National Academy of Sciences (1983). The Academy defines a *small dam* as having 50–1000 acre-feet of storage or being 25–40 ft high, an *intermediate dam* as having 1000–50,000 acre-ft of storage or being 40–100 ft high, and a *large dam* as having more than 50,000 acre-ft of storage or being more than 100 ft high. In general, there would be considerable loss of life and extensive damage if a major structure failed. In the case of an intermediate structure, a small loss of life would be possible and the damage would be within the financial capability of the owner. For minor structures, there generally would be no loss of life, and the damage would be of the same magnitude as the cost of replacing or repairing the structure.

**Table 10.3.1** Generalized Design Criteria for Water-Control Structures

| Type of Structure | Return Period (Years) | ELV |
|---|---|---|
| Highway culverts | | |
| Low traffic | 5–10 | — |
| Intermediate traffic | 10–25 | — |
| High traffic | 50–100 | — |
| Highway bridges | | |
| Secondary system | 10–50 | — |
| Primary system | 50–100 | — |
| Farm drainage | | |
| Culverts | 5–50 | — |
| Ditches | 5–50 | — |
| Urban drainage | | |
| Storm sewers in small cities | 2–25 | — |
| Storm sewers in large cities | 25–50 | — |
| Airfields | | |
| Low traffic | 5–10 | — |
| Intermediate traffic | 10–25 | — |
| High traffic | 50–100 | — |
| Levees | | |
| On farms | 2–50 | — |
| Around cities | 50–200 | — |
| Dams with no likelihood of loss of life (low hazard) | | |
| Small dams | 50–100 | — |
| Intermediate dams | 100 + | — |
| Large dams | — | 50–100% |
| Dams with probable loss of life (significant hazard) | | |
| Small dams | 100 + | 50% |
| Intermediate dams | — | 50–100% |
| Large dams | — | 100% |
| Dams with high likelihood of considerable loss of life (high hazard) | | |
| Small dams | — | 50–100% |
| Intermediate dams | — | 100% |
| Large dams | — | 100% |

*Source*: Chow et al. (1988).

### 10.3.2 Hydrologic Design Level (Return Period)

A *hydrologic design level* on the design scale is the magnitude of the hydrologic event to be considered for the design of a structure or project. As it is not always economical to design structures and projects for the estimated limiting values, the ELV is often modified for specific design purposes. The final design value may be further modified according to engineering judgment and the experience of the designer or planner. Table 10.3.1 presents generalized criteria for water-control structures. A large number of the structures are designed using return periods.

An extreme hydrologic event is defined to have occurred if the magnitude of the event $X$ is greater than or equal to some level $x_T$, i.e., $X \geq x_T$. The *return period* $T$ of the event $X = x_T$ is the expected value of the *recurrence interval* (time between occurrences). The expected value $E(\ )$ is the average value measured over a very large number of occurrences. Consequently, the return period of a hydrologic event of a given magnitude is defined as the *average recurrence interval* between events that equal or exceed a specified magnitude.

### 10.3.3 Hydrologic Risk

The *probability of occurrence* $P(X \geq x_T)$ of the hydrologic event $(X \geq x_T)$ for any observation is the inverse of the return period, i.e.,

$$P(X \geq x_T) = \frac{1}{T} \tag{10.3.1}$$

For a 100-year peak discharge, the probability of occurrence in any given year is $P(X \geq x_{100}) = 1/100 = 0.01$.

The probability of nonexceedance is

$$P(X < x_T) = 1 - \frac{1}{T} \tag{10.3.2}$$

Because each hydrologic event is considered independent, the *probability of nonexceedance* for $n$ years is

$$P(X < x_T \text{ each year for } n \text{ years}) = \left(1 - \frac{1}{T}\right)^n$$

The complement, the *probability of exceedance* at least once in $n$ years, is

$$P(X \geq x_T \text{ at least once in } n \text{ years}) = 1 - \left(1 - \frac{1}{T}\right)^n$$

which is the probability that a $T$-year return period event will occur at least once in $n$ years. This is also referred to as the *natural, inherent,* or *hydrologic risk of failure* $\bar{R}$:

$$\bar{R} = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - [1 - P(X \geq x_T)]^n \tag{10.3.3}$$

where $n$ is referred to as the expected life of the structure. The hydrologic risk relationship is plotted in Figure 10.3.2.

**EXAMPLE 10.3.1** Determine the hydrologic risk of a 100-year flood occurring during the 30-year service life of a project.

**SOLUTION** Use equation (10.3.3) to determine the risk:

$$\bar{R} = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - (1 - 1/100)^{30} = 0.26$$

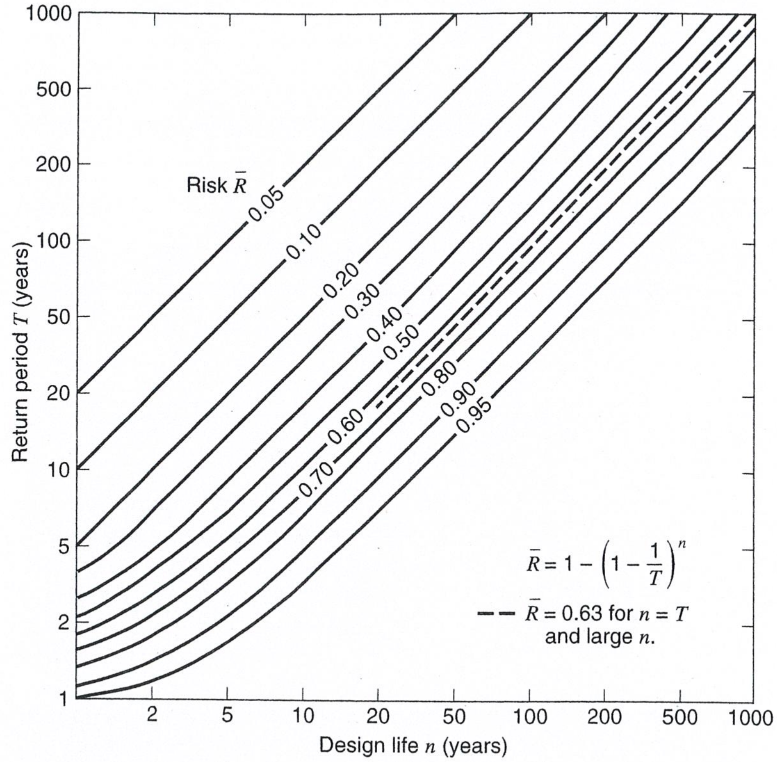



**Figure 10.3.2** Risk of at least one exceedance of the design event during the design life (from Chow et al. (1988)).

### 10.3.4 Hydrologic Data Series

Figure 10.3.3*a* shows all the data available (that have been collected) for a hydrologic event. This represents a *complete-duration series*. A *partial-duration series* includes data that are selected so that their values are greater than some base value. An *annual-exceedance series* has a base value so that the number of values in the series is equal to the number of years of record. Figure 10.3.3*b* illustrates the annual exceedance series. An *extreme-value series* consists of the largest or smallest values occurring in each of the equally long time intervals of the record. If the time interval length is one year, the series is an *annual series*. An *annual maximum* series over the largest values in each respective year (Figure 10.3.3*c*) consists of the largest annual values and an *annual minimum series* consists of the smallest annual values in each of the respective years. Figure 10.3.4 illustrates the annual-exceedence series and the annual maximum series of the hypothetical data in Figure 10.3.3.

The return periods for annual exceedance series $T_E$ are related to the corresponding annual maximum series return period $T$ by (Chow, 1964)

$$T_E = \left[\ln\left(\frac{T}{T-1}\right)\right]^{-1} \tag{10.3.4}$$

SOURCE: Mays, Water Resources Engineering, Wiley, 2005 (ISBN 0-471-70524-1)

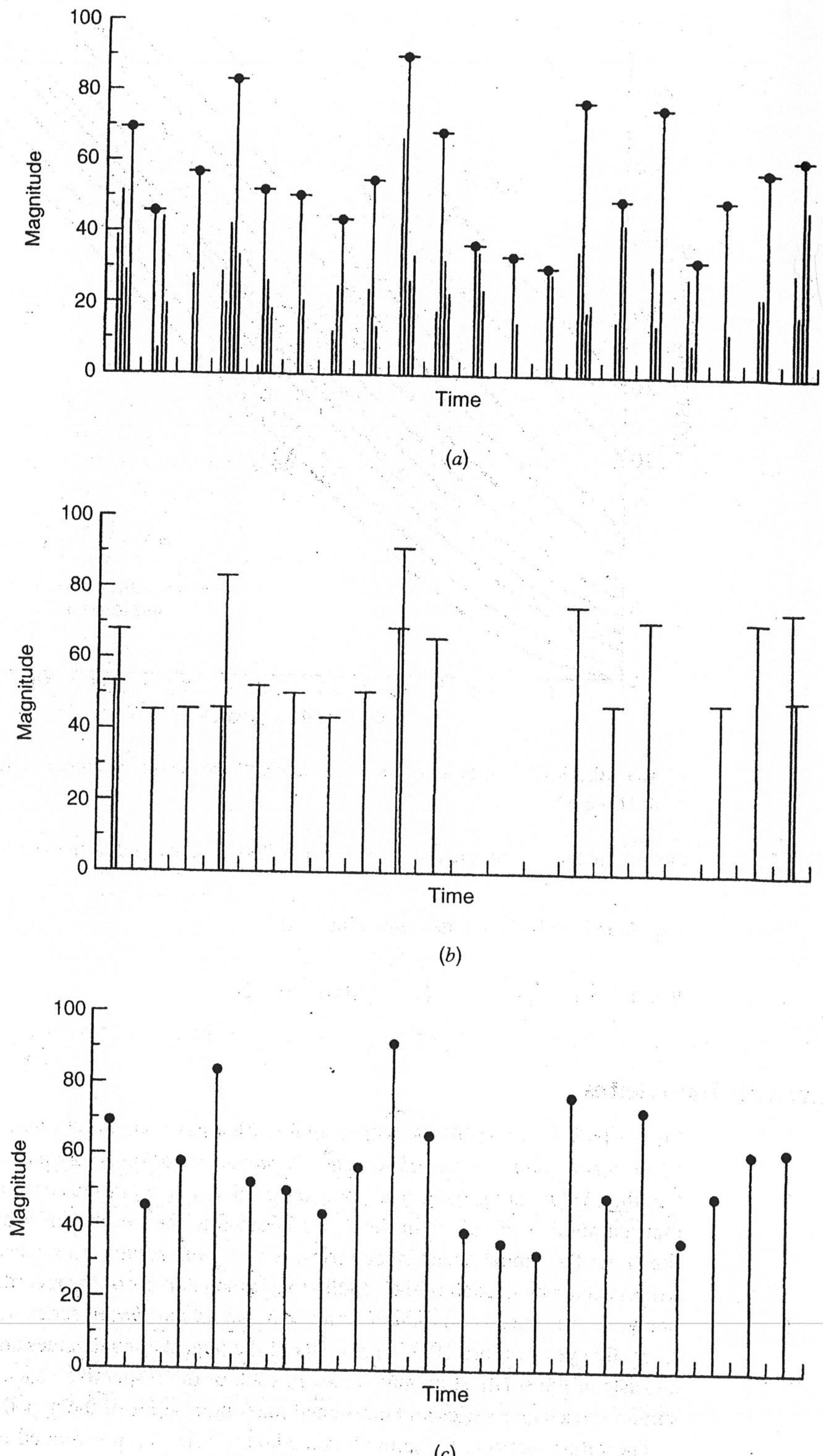



**Figure 10.3.3** Hydrologic data arranged by time of occurrence. (*a*) Original data: $N = 20$ years. (*b*) Annual exceedances. (*c*) Annual maxima (from Chow (1964)).

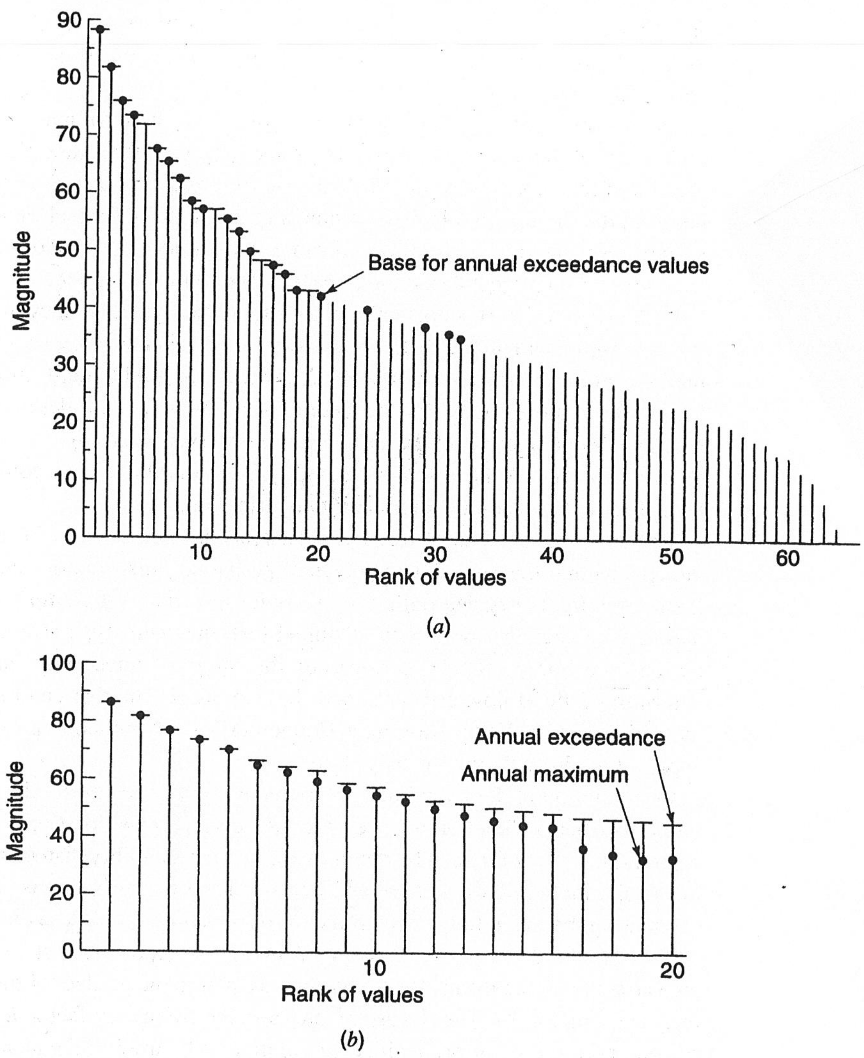



**Figure 10.3.4** Hydrologic data arranged in the order of magnitude. (*a*) Original data. (*b*) Annual exceedance and maximum values (from Chow (1964)).

**EXAMPLE 11.1**

A culvert has been designed for a 50-year exceedance interval. What is the probability that exactly one flood of the design capacity will occur in the 100-year lifetime of the structure?

**SOLUTION** $n = 100$, $k = 1$. Exceedance probability, $P = \frac{1}{50} = 0.02$. From eq. (11.3),

$$f_x\{1 \text{ event in 100 years}\} = C_1^{100} P^1 (1-P)^{100-1}$$

$$= \frac{100!}{1!(100-1)!}(0.02)^1 (1-0.02)^{99}$$

$$= 0.27$$

**EXAMPLE 11.2**

In Example 11.1, what is the probability that the culvert will experience the design flood one or more times (at least once) in its lifetime?

**SOLUTION** From eq. (11.4),

$$f_x\{\text{at least once in 100 years}\} = 1-(1-0.02)^{100}$$

$$= 0.87$$

**EXAMPLE 11.3**

The spillway of a dam has a service life of 75 years. A risk of 5% for the failure of the structure (exceeding of the flood capacity) has been considered acceptable. For what return period should the spillway capacity be designed?

**SOLUTION**

$$f_x = \frac{5}{100} = 0.05$$

$$n = 75$$

From eq. (11.4),

$$f_x = 1 - (1 - P)^n$$

$$0.05 = 1 - (1 - P)^{75}$$

$$P = 0.000684$$

$$T = \frac{1}{P} = 1460 \text{ years}$$