Fundamentals of Hydraulic Engineering Systems

Fifth Edition

Chapter 2b

Water Pressure and Pressure Forces

Hydrostatic Forces - Curved Surfaces (1 of 4)

Visualization and Analysis

Consider the curved gate AB.

Visualization: Draw the pressure prism acting on the gate.

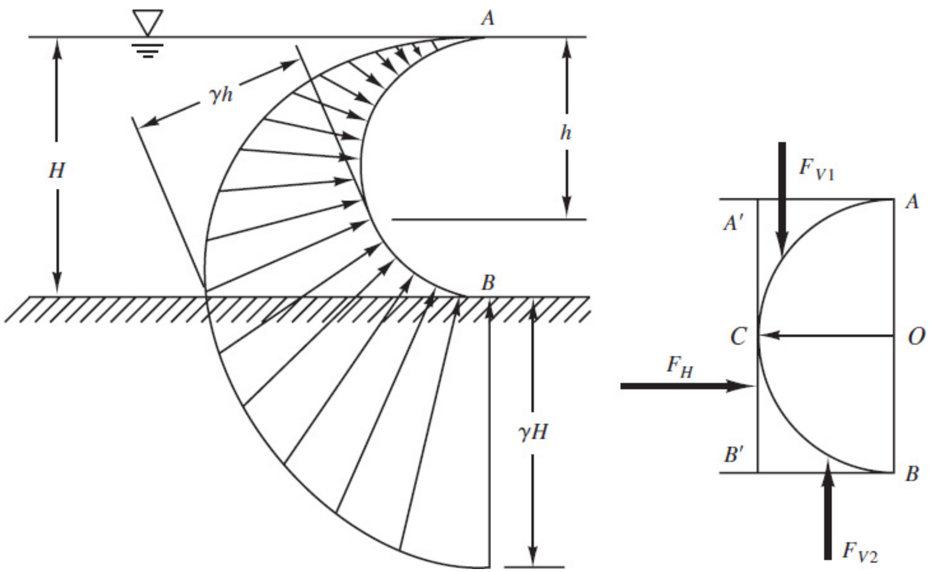
Note: Pressures will act normal to the gate and increase with depth.

Question: Will the resultant force have *x* and *y* components?

Analysis: Solve for horizontal & vertical components separately.

How would you obtain the forces

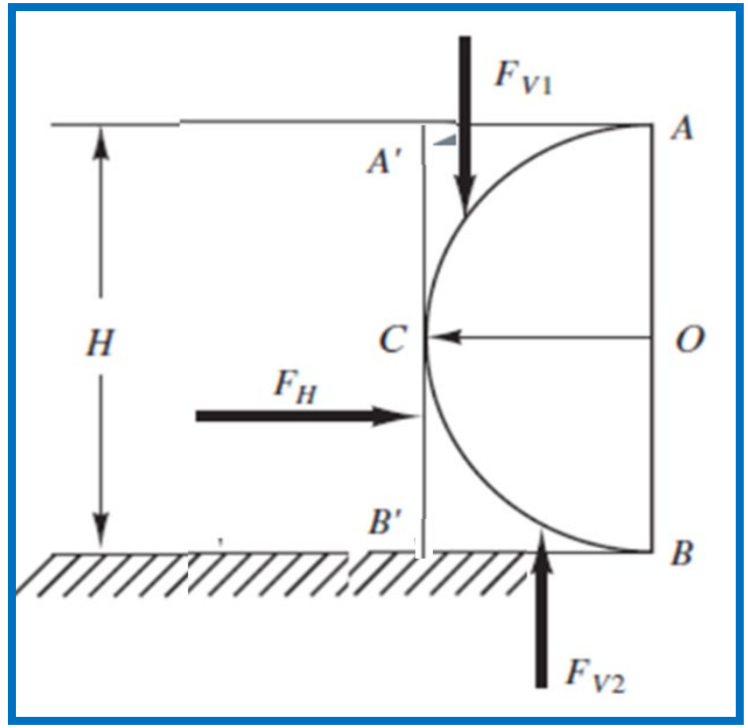
Figure 2.14



Hydrostatic Forces - Curved Surfaces

Example Problem

Find the component forces on the semicircular gate AB (assume a unit width).



Horizontal Component (F_H).

$F_H = \gamma \bar{h} A$ **Note:** The area for curved surfaces is the vertical projection of the surface area. (A′B′, in this case a rectangle.)

$$F_H = \gamma \bar{h} A = \gamma (H/2)(H \cdot 1) = \gamma (H)^2 / 2$$

Location : $h_p = [I_o / (A\bar{h})] + \bar{h} = [(H^3/12)/(H * H/2)] + H/2 = 2H/3$

Hydrostatic Forces - Curved Surfaces (3 of 4)

Example Problem

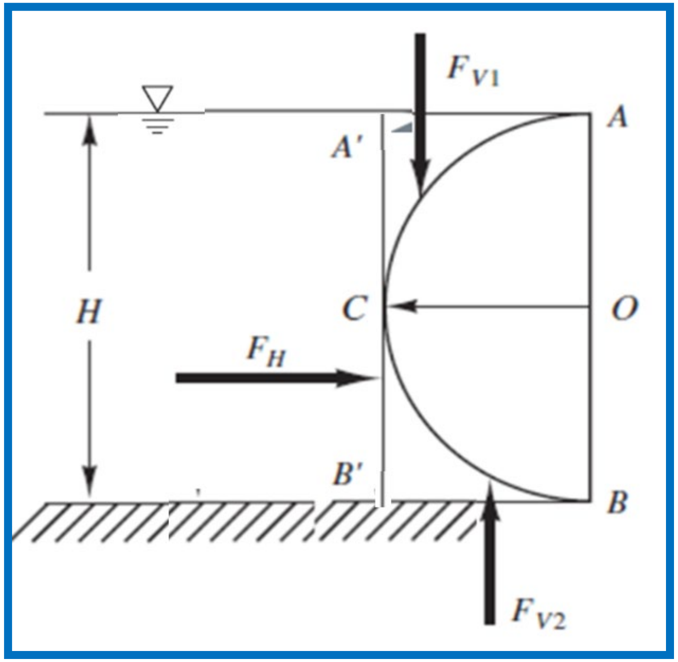
Vertical Component (F_{V1}). $\uparrow +$

$F_{V1} = W = \gamma(Vol)$ **Note:** The force is equal to the weight of the water column above the surface.

$$F_{V1} = -\gamma[(H/2 * H/2) - \{(1/4)(\pi(H/2)^2\}]$$

$$F_{V1} = -\gamma[(H^2/4) - \{\pi(H)^2/16\}]$$

$F_{V2} = \gamma(Vol)$ **Note:** This force is equal to the weight of the imaginary water column above.



Hydrostatic Forces - Curved Surfaces (4 of 4)

Find F(total) and *θ* w/vector analysis.

$$F_{V2} = \gamma[\left(H/2 * H/2\right) + \{(1/4)(\pi\left(H/2\right)^2\}] = \gamma[(H^2/4) + \{\pi\left(H\right)^2/16\}]$$

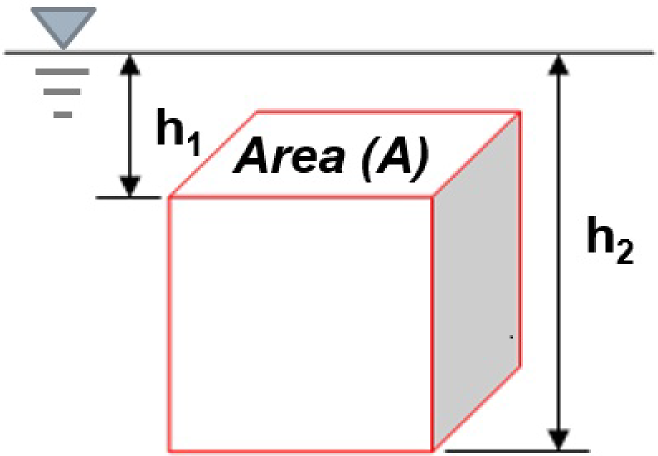
$$F_V = F_{V1} + F_{V2} = \gamma\left(\pi/8\right)H^2$$

Buoyancy and Archimedes Principle (1 of 2)

Find the hydrostatic force on the top and bottom of the box.

$F_{Top} = -pA = -\gamma h_1 A$ **(Positive is up.)**

$F_{Bottom} = pA = \gamma h_2 A$



Find the net (Buoyant) force:

$F_V = \gamma(h_2 - h_1)A;$ **But** $(h_2 - h_1)A = Vol;$ **Therefore,** $F_V = \gamma\left(Vol\right)$

Archimedes Principle: The buoyant force on a submerged object is equal to the weight of the water displaced.

Question: Why is it easier to float in the ocean than a lake?

Flotation Stability (1 of 2)

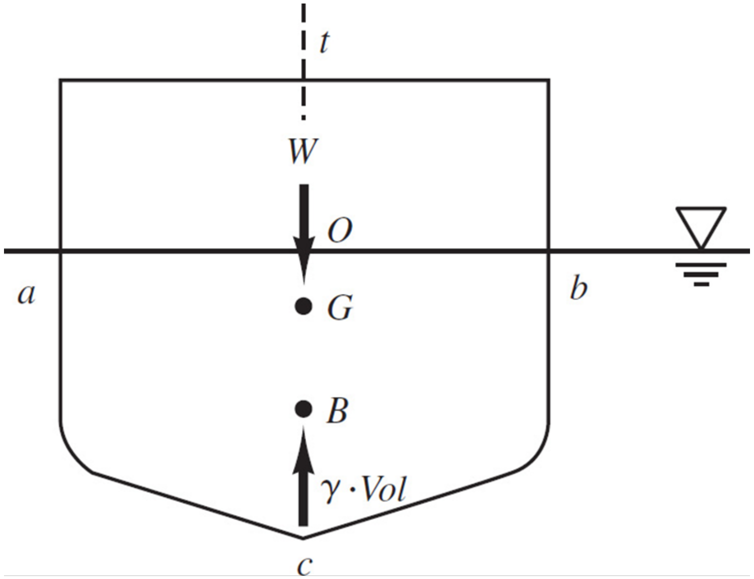
Equilibrium Position

Definitions:

W = weight, G → location of weight (center of gravity)

$\gamma \cdot$ Vol $=$ buoyant force

B → location of buoyant force (center of buoyancy);
i.e., the center of gravity of the liquid volume displaced by the floating body



X-section: ship's hull

Undisturbed Position:

W = γ·Vol → No moment (G and B are on same vertical line)

Flotation Stability (2 of 2)

Non-equilibrium Position

Disturbance (wind or waves)

The heal (or list) angle is θ.

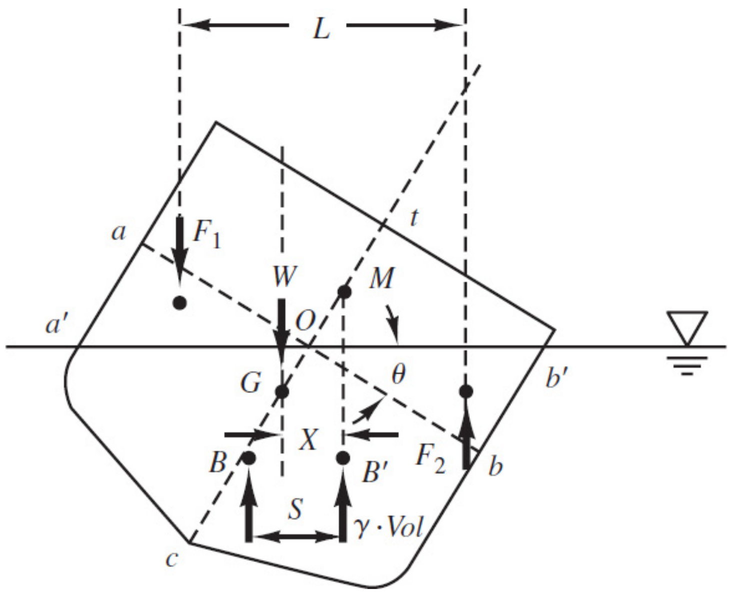
G location doesn't change.

B' is new buoyancy location.

Result: Moment (or couple)

Righting Moment $(M = W \cdot X)$

(i.e., will resist overturning)



Definitions: $M \rightarrow$ metacenter $GM \rightarrow$metacentric height;

Equations : $M = W \cdot X = W(GM)\sin\theta$,

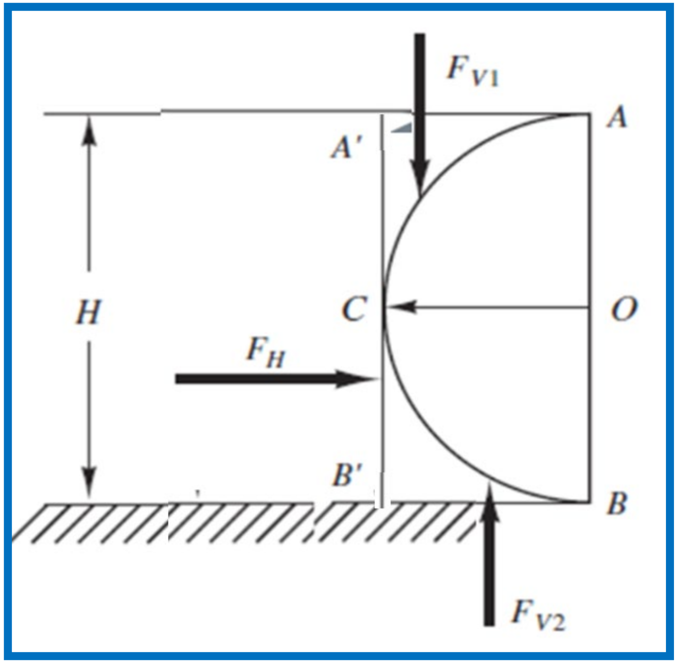
$GM = MB \pm GB = I_0 / Vol \pm GB$; where Vol = submerged volume;

I_0 = moment of inertia of waterline x-section area

Buoyancy and Archimedes Principle (2 of 2)

Example Problem

Find the buoyant force on the semicircular gate AB (unit width) and compare it to the net vertical force $(F_V = F_{V1} + F_{V2})$ that we computed previously.



Homework Problems:

Buoyancy in Action

The Welland Canal connects Lake Ontario and Lake Erie through a series of locks and dams allowing ships to bypass Niagara Falls.

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