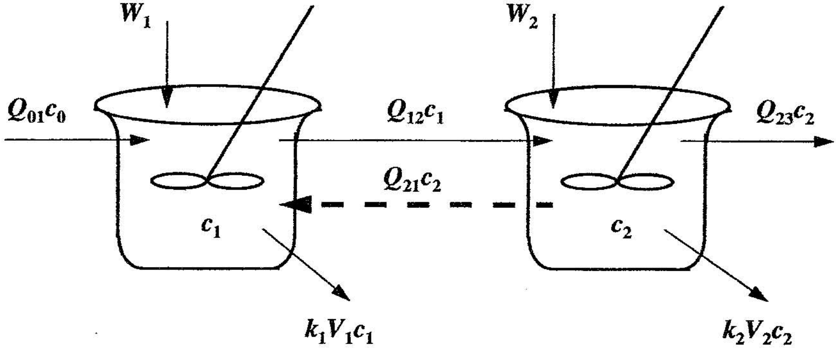
Lecture 6
Feedback Systems of Reactors



Mass Balance Equations

$$V_1 \,(dc_1/dt) = W_1 + Q_{0,1}c_0 - Q_{1,2}c_1 - k_1 V_1 c_1 + Q_{21}c_2$$

$$V_2 \,(dc_2/dt) = W_2 + Q_{12}c_1 - Q_{21}c_2 - Q_{23}c_2 - k_2 V_2 c_2$$

Collected terms: $a_{11}c_1 + a_{12}c_2 = W_1$ $(W_1 \leftarrow W_1 + Q_{01}C_O)$

$$a_{21}c_1 + a_{22}c_2 = W_2$$

Algebraic manipulation:

$$c_1 = W_1/\,(a_{11} - (a_{21}a_{12}/a_{22})) + W_2/\,(a_{21} - (a_{11}a_{22}/a_{12}))$$

$$c_2 = W_1/\,(a_{12} - (a_{11}a_{22}/a_{21})) + W_2/\,(a_{22} - (a_{21}a_{12}/a_{11}))$$

Large Systems of Reactors

For 3 coupled reactors the linear algebraic equations are:

$$a_{11}c_1 + a_{12}c_2 + a_{13}c_3 = W_1$$

$$a_{21}c_1 + a_{22}c_2 + a_{23}c_3 = W_2$$

$$a_{31}c_1 + a_{32}c_2 + a_{33}c_3 = W_3$$

Matrix Algebra

"Matrix algebra is used to solve complex systems"

$$[A]\{C\} = \{W\}$$

$$\{C\} = [A]^{-1}\{W\}$$

$$\{Response\} = [Interactions]\{stimuli\}$$

Time-Variable Response

Two lakes in series with no loads at SS condition:

$$dc_1/dt = \alpha_{11}c_1 - \alpha_{12}c_2$$
$$dc_2/dt = \alpha_{21}c_1 - \alpha_{22}c_2$$

where

$$c_1 = c_{1f}e^{-\lambda ft} + c_{1s}e^{-\lambda st}$$
$$c_2 = c_{2f}e^{-\lambda ft} + c_{2s}e^{-\lambda st}$$

and

λf and λs are eigenvalues.

Closed Systems

- First order reaction takes place in a batch reactor
- Mass balance

$$dc_a/dt = -k_{ab}c_a + k_{ba}c_b$$
$$dc_b/dt = k_{ab}c_a - k_{ba}c_b$$

$$c_a = c_{ao}\, e^{-(kab+kba)t} + \hat{c}_a\,(1\text{-}e^{-(kab+kba)t})$$
$$c_b = c_{bo}\, e^{-(kab+kba)t} + \hat{c}_b\,(1\text{-}e^{-(kab+kba)t})$$

Note $\hat{c}$ = average c

Open Systems (as CSTRs)

Mass balances

$$dc_a/dt = (Q/V)c_{a,in} - (Q/V)c_a - k_{ab}c_a + k_{ba}c_b$$

$$dc_b/dt = (Q/V)c_{b,in} - (Q/V)c_b - k_{ab}c_a - k_{ba}c_b - k_b c_b$$