Supporting Handout to Chapter 2 (CAHE, 9th Edition)

CWR5535C – Advanced Modeling Applications in Water Resources Engineering

Spring 2024

Main References:

Professor H. R. Fuentes Personal Notes, 2024

Gupta, R. S., Hydrology & Hydraulic Systems, Waveland Press, Inc., ISBN: 1-4786-3091-4, Long Grove, IL, 2017

Precipitation - Runoff Relationship

Importance: a) Hydrologic/Hydraulic Design
b) " " Analysis

Method (approach, "model")	Complexity Level	Assumptions	Limitations (Examples)
Rational "Formula" ($Q = C_f C i A$)	↓	↑	$A < 1\ mi^2$ (Robertson et al., 1998) $A < 0.015 - 4.6\ mi^2$ (Other references) - Small watersheds - Short t_c.
NRCS [or Technical Release TR-55 (1975, 1986)]			$A < 5-10\ mi^2$ (Robertson et al., 1998) Conservative if storm is of <u>long</u> duration
Unit Hydrograph - Based on actual data (actual hydrographs) - Synthetic			$A < 40-50\ mi^2$ (thunderstorms) $A < 2{,}000 - 3{,}000\ mi^2$ (frontal storms,

United States
Department of
Agriculture

Natural
Resources
Conservation
Service

Conservation
Engineering
Division

Technical
Release 55

June 1986

Urban Hydrology for Small Watersheds

TR-55

To show bookmarks which navigate through the document.

Click the show/hide navigation pane button ▤ , and then click the bookmarks tab. It will navigate you to the contents, chapters, rainfall maps, and printable forms.

Contents

Figures

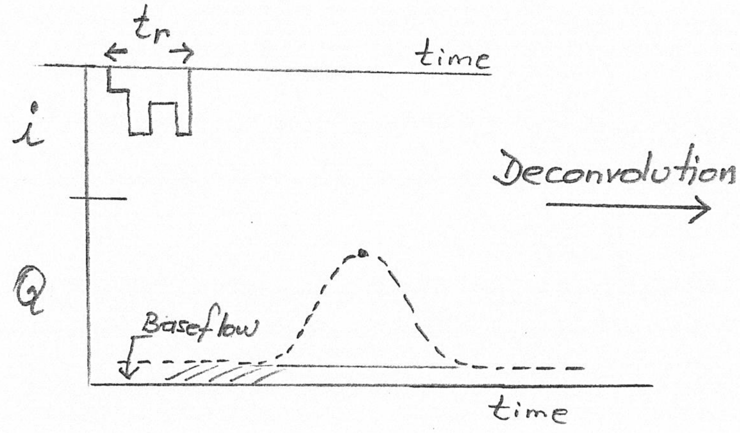
Tables

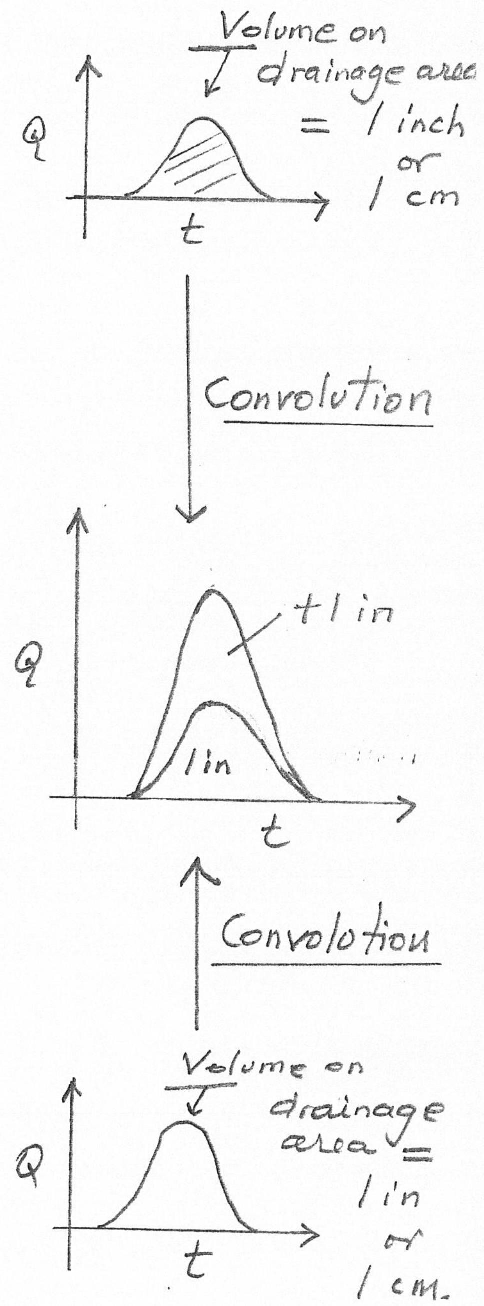
Exhibits

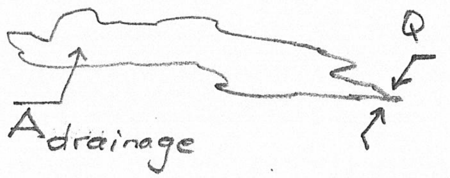
Worksheets

UNIT HYDROGRAPH
Development & Application

a) From actual hydrographs:







b) Synthetic Unit Hydrographs
(develop based on drainage area characteristics)
Examples: Snyder's UH
NRCS' UH

UH Basic Principles

Figure 9.9 Principles of the unit hydrograph: (a) unit hydrograph; (b) runoff hydrograph for two units of precipitation of duration t_r; (c) runoff hydrograph from unit precipitation for two consecutive periods of duration t_r.

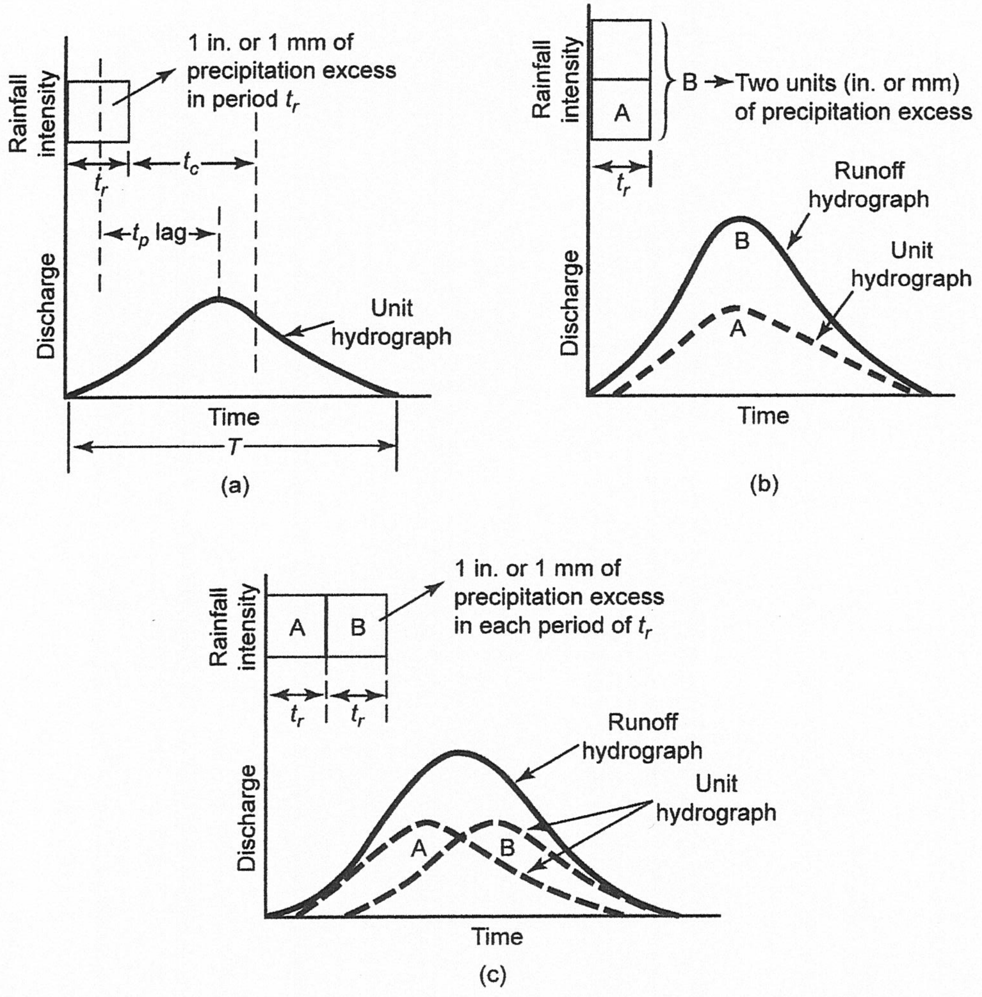
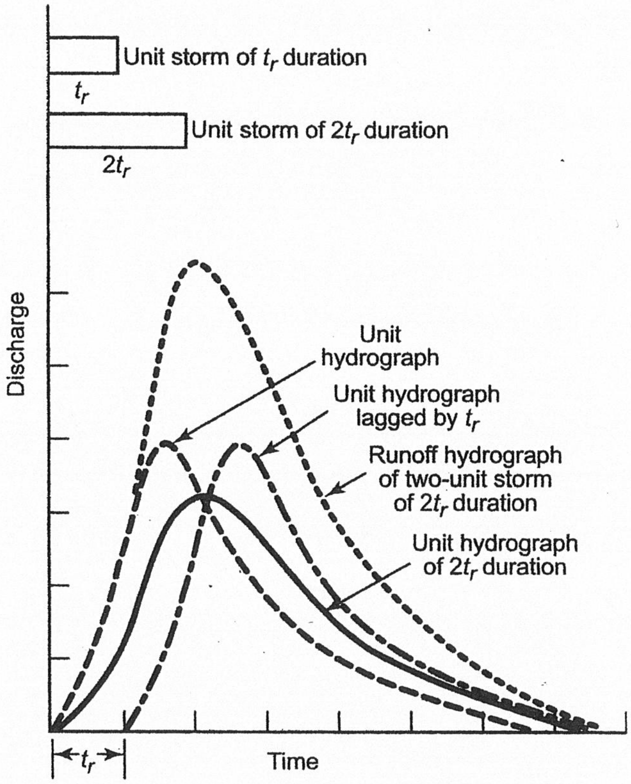


Figure 9.15 Lagging procedure to convert unit hydrograph duration.



$$\text{UH of } nt_r \text{ duration} = \frac{\begin{array}{c}\text{sum of } n\text{, UH of } t_r \text{ duration}\\ \text{each lagged by } t_r \text{ time}\end{array}}{n} \quad [\mathrm{L}^3\mathrm{T}^{-1}] \tag{9.8}$$

EXAMPLE 9.5

The following unit hydrograph results from a 2-hour storm. Determine the hourly ordinates of a 6-hour unit hydrograph.

Time (hr)	0	1	2	3	4	5	6
Q (m^3/s)	0	1.42	8.50	11.30	5.66	1.45	0

SOLUTION See Table 9.8.

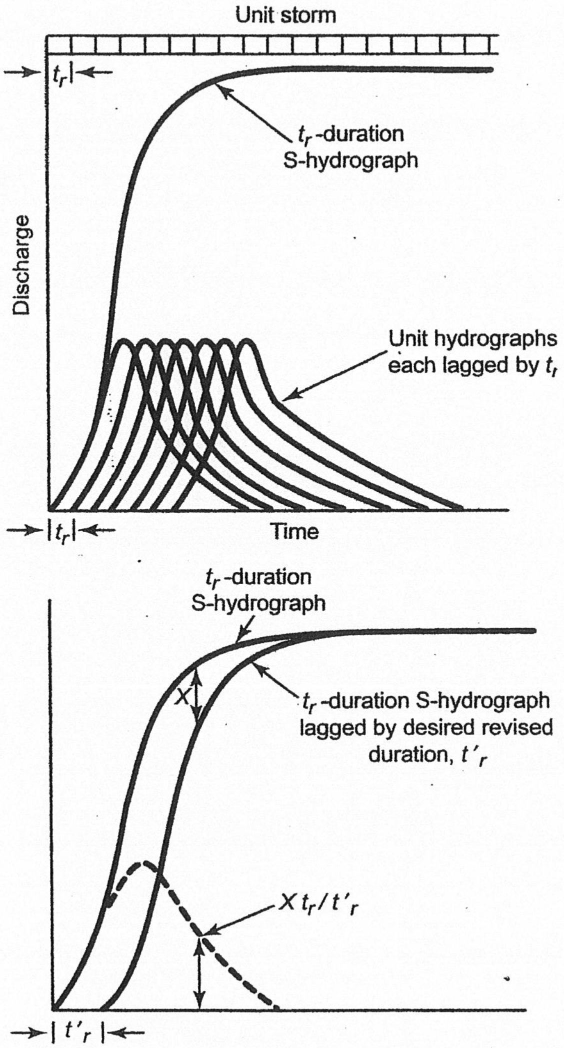
$$t_r = 2 \text{ hr}$$

$$n = \frac{\text{6-hr duration}}{\text{2-hr duration}} = 3$$

Table 9.8 Conversion of Unit Hydrograph Duration by Lagging

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Three 2-hr Hydrographs Each Lagged by 2 hr				
Time (hr)	2-hr Unit Hydrograph (m^3/s)	1 × UH	1 × UH	1 × UH	Total	6-hr Unit Hydrograph (m^3/s) (col. 6/3.0)
0	0	0			0	0
1	1.42	1.42			1.42	0.47
2	8.50	8.50	0		8.50	2.83
3	11.30	11.30	1.42		12.72	4.24
4	5.66	5.66	8.50	0	14.16	4.72
5	1.45	1.45	11.30	1.42	14.17	4.72
6	0	0	5.66	8.50	14.16	4.72
7			1.45	11.30	12.75	4.25
8			0	5.66	5.66	1.89
9				1.45	1.45	0.48
10				0	0	0

Figure 9.16 Illustration of the S-curve.



EXAMPLE 9.6

Solve Example 9.5 by the S-curve method.

SOLUTION Computations are shown in Table 9.10.

Table 9.10 Computation of 2-Hour S-Curve and 6-Hour Unit Hydrograph

(1) Time (hr)	(2) 2-hr Unit Hydrograph (m^3/s)	(3) S-Curve Addition	(4) 2-hr S-Curve	(5) 2-hr S-Curve Lagged by 6-hr	(6) S-Curve Difference	(7) 6-hr Unit Hydrograph (col. 6 × 2/6)
0	0	+0	0		0	0
1	1.42	+0	1.42		1.42	0.47
2	8.50	+0	8.50		8.50	2.83
3	11.30	+1.42	12.72		12.72	4.24
4	5.66	+8.50	14.16		14.16	4.72
5	1.45	+12.72	14.17		14.17	4.72
6	0	14.16	14.16	0	14.16	4.72
7		14.17	14.17	1.42	12.75	4.25
8		14.16	14.16	8.50	5.66	1.89
9		14.17	14.17	12.72	1.45	0.48
10		14.16	14.16	14.16	0	0
11		14.17	14.17	14.17	0	0

Two Synthetic Unit Hydrographs

9.11.1 Snyder's Method

The four parameters—lag time, peak flow, time base, and standard duration—of rainfall excess for the unit hydrograph have been related to the physical geometry of the basin by the following relations:

$$t_p = C_t (LL_C)^{0.3} \quad \text{[unbalanced]} \tag{9.9}$$

$$Q_p = \frac{C_p A}{t_p} \quad \text{[unbalanced]} \tag{9.10}$$

$$T = 3 + \frac{t_p}{8} \quad \text{[T]} \tag{9.11}$$

$$t_D = \frac{t_p}{5.5} \quad \text{[T]} \tag{9.12}$$

When the duration of rainfall excess, t_r, is other than the standard duration, t_D, the following adjustments in lag time and peak discharge are made:

$$t_{pR} = t_p + 0.25\,(t_r - t_D) \quad \text{[T]} \tag{9.13}$$

$$Q_{pR} = Q_p \frac{t_p}{t_{pR}} \quad [\mathrm{L^3T^{-1}}] \tag{9.14}$$

where

t_D = standard duration of rainfall excess, hours

t_r = duration of rainfall excess other than standard duration adopted in the study, hours

t_p = lag time from midpoint of rainfall excess duration, t_D, to peak of the unit hydrograph, hours

t_{pR} = lag time from midpoint of duration, t_r, to the peak of the unit hydrograph, hours

T = time base of unit hydrograph, days

Q_p = peak flow for standard duration, t_D

Q_{pR} = peak flow for duration, t_r

L_C = stream mileage from the outlet to a point opposite the basin centroid

L = stream mileage from the outlet to the upstream limits of the basin

A = drainage area, mi^2 or km^2

C_t = coefficient representing slope of the basin;

varies from 1.8 to 2.2 for distance in miles, or from 1.4 to 1.7 for distance in kilometers;

Taylor and Schwarz state that C_t equals $0.6/\sqrt{S}$ for distance in miles, S being the basin slopes

C_p = coefficient indicating the storage capacity;

varies from 360 to 440 for English units, and from 0.15 to 0.19 for metric units

If the ungaged basin and the gaged basin are located in close proximity to each other within a region, the coefficients C_t and C_p are computed from the data of the gaged basin. The coefficients so obtained are used in the preceding equations to construct the unit hydrograph for the ungaged basin. Otherwise, generalized values are used for the coefficients.

A unit hydrograph is sketched, from the lag time, peak discharge, and time base computed from eqs. (9.9) through (9.14), to represent a unit runoff amount (area under the graph). Equation (9.11) usually gives long base length for small to medium basins. The following Corps of Engineers formulas give additional assistance in plotting time width, W_{50}, in hours, at the discharge point equal to 50% of the peak discharge, and the width, W_{75}, in hours, at the discharge point equal to 75% of the peak flow.

$$W_{50} = \frac{770A^{1.08}}{Q_{pR}^{1.08}} \quad \text{(English units)} \quad \text{[unbalanced]} \tag{9.15a}$$

or

$$W_{50} = \frac{0.23A^{1.08}}{Q_{pR}^{1.08}} \quad \text{(metric units)} \quad \text{[unbalanced]} \tag{9.15b}$$

and

$$W_{75} = \frac{440A^{1.08}}{Q_{pR}^{1.08}} \quad \text{(English units)} \quad \text{[unbalanced]} \tag{9.16a}$$

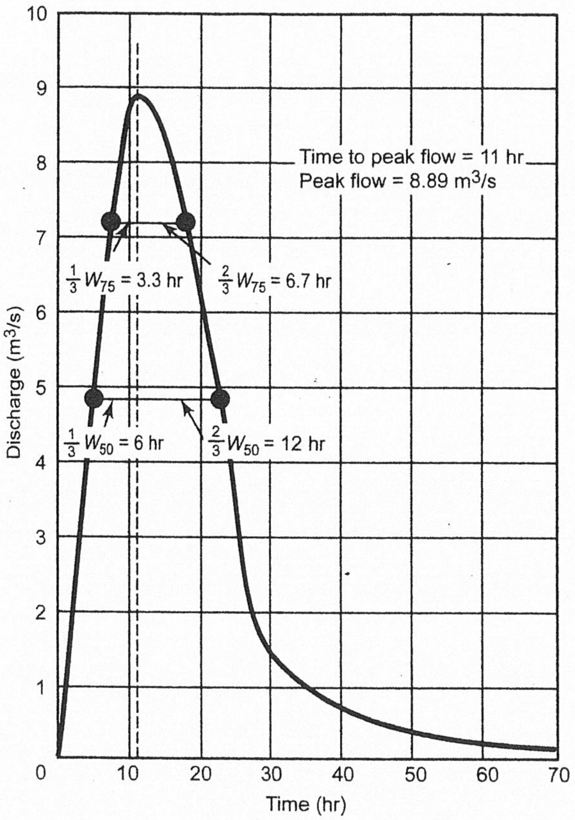
or

$$W_{75} = \frac{0.13A^{1.08}}{Q_{pR}^{1.08}} \quad \text{(metric units)} \quad \text{[unbalanced]} \tag{9.16b}$$

In eqs. (9.15a) and (9.16a), A is in mi^2 and Q in cfs, and in eqs. (9.15b) and (9.16b), A is in km^2 and Q in m^3/s.

As a rule of thumb, the widths W_{50} and W_{75} are proportioned each side of the unit hydrograph peak in the ratio 1:2, with the short side on the left of the synthetic unit hydrograph.

Figure 9.17 Synthetic unit hydrograph by Snyder's method.



EXAMPLE 9.7

For a basin of 500 km^2 having $L = 25$ km and $L_C = 10$ km, derive the 4-hour unit hydrograph. Assume that $C_t = 1.6$ and $C_p = 0.16$.

SOLUTION

1. $t_r = 4$ hr (given). From eq. (9.9),

$$t_p = 1.6(25 \times 10)^{0.3} = 8.38 \text{ hr}$$

2. From eq. (9.10),

$$Q_p = \frac{0.16(500)}{8.38} = 9.55 \text{ m}^3/\text{s}$$

3. From eq. (9.11),

$$T = 3 + \frac{8.38}{8} = 4.05 \text{ days or } 97 \text{ hr}$$

4. From eq. (9.12),

$$t_D = \frac{8.38}{5.5} = 1.5 \text{ hr}$$

5. From eq. (9.13),

$$t_{pR} = 8.38 + 0.25(4 - 1.5) = 9 \text{ hr}$$

6. From eq. (9.14),

$$Q_{pR} = \frac{9.55(8.38)}{9.0} = 8.89 \text{ m}^3/\text{s}$$

7. Time from beginning to peak,

$$p_r = \frac{t_r}{2} + t_{pR} = 2 + 9 = 11 \text{ hr}$$

8. From eq. (9.15b),

$$W_{50} = \frac{0.23(500)^{1.08}}{(8.89)^{1.08}} = 18 \text{ hr}$$

9. From eq. (9.16b),

$$W_{75} = \frac{0.13(500)^{1.08}}{(8.89)^{1.08}} = 10 \text{ hr}$$

The unit hydrograph has been sketched in Figure 9.17.

9.11.2 Natural Resources Conservation Service (NRCS) Method

The NRCS employs an average dimensionless hydrograph developed from an analysis of a large number of unit hydrographs from field data of various-sized basins in different geographic locations.

This dimensionless hydrograph has its ordinate values of discharge expressed as the dimensionless ratio with the peak discharge and its abscissa values of time interval as the dimensionless ratio with the period of rise (time from beginning to the peak flow). The ratios for the NRCS dimensionless unit hydrograph are given in Table 9.11.

The unit hydrograph ordinates for different time periods can be obtained from Table 9.11. However, to use this table, the values of P_r and Q_p are required, which are computed as follows:

$$Q_p = \frac{484A}{P_r} \quad \text{(English units)} \quad \text{[unbalanced]} \tag{9.17a}$$

or

$$Q_p = \frac{0.208A}{P_r} \quad \text{(metric units)} \quad \text{[unbalanced]} \tag{9.17b}$$

$$P_r = \frac{t_r}{2} + t_p \quad [\mathrm{T}] \tag{9.18}$$

The time lag, t_p, is computed by eq. (9.9) or by a regional empirical relation, or by the NRCS equation involving the NRCS curve number.

Table 9.11 Ratios for the NRCS Dimensionless Unit Hydrograph

Time Ratio, t/P_r	Hydrograph Discharge Ratio, (Q/Q_p)
0	0
0.1	0.030
0.2	0.100
0.3	0.190
0.4	0.310
Ex. 0.5 = $t_{0.5}/P_r \therefore t_{0.5} = 0.5 \times \overset{\text{Eq. 9.18}}{P_r}$	0.470 = $Q/Q_p \therefore Q = 0.470\,\overset{\text{Eq. 9.17b}}{Q_p}$
0.6	0.660
0.7	0.820
0.8	0.930
0.9	0.990
1.0	1.000
1.1	0.990
1.2	0.930
1.3	0.860
1.4	0.780
1.5	0.680
1.6	0.560
1.8	0.390
2.0	0.280
2.2	0.207
2.4	0.147
2.6	0.107
2.8	0.077
3.0	0.055
3.5	0.025
4.0	0.011
4.5	0.005
5.0	0.000

Source: NRCS (formerly Soil Conservation Service), 1972.

EXAMPLE 9.8

Solve Example 9.7 by the NRCS method.

SOLUTION

1. $t_p = 8.38$ hr, from eq. (9.9) computed in Example 9.7.
2. From eq. (9.18), $P_r = 4/2 + 8.38 = 10.38$ hr ≈ 10.5 hr.
3. From eq. (9.17b),

$$Q_p = \frac{0.208(500)}{10.5} = 9.90 \text{ m}^3/\text{s}$$

4. Using Table 9.11, the hydrograph ordinates are given in Table 9.12.

Table 9.12 Synthetic Unit Hydrograph by NRCS Method

(1) t/P_r	(2)[a] t (hr)	(3) Q/Q_p (from Table 9.11)	(4)[b] Q (m^3/s)
0	0	0	0
0.2	2.1	0.100	0.99
0.5	5.25	0.470	4.65
0.8	8.4	0.930	9.21
1.0	10.5	1.00	9.90
1.5	15.75	0.680	6.73
2.0	21.0	0.280	2.77
3.0	31.5	0.055	0.54
4.0	42.0	0.011	0.11
5.0	52.5	0.000	0.00

[a] Col. 2 = col. 1 × P_r

[b] Col. 4 = col. 3 × Q_p

STATE OF FLORIDA DEPARTMENT OF TRANSPORTATION

DRAINAGE MANUAL

OFFICE OF DESIGN, DRAINAGE SECTION JANUARY 2015
TALLAHASSEE, FLORIDA

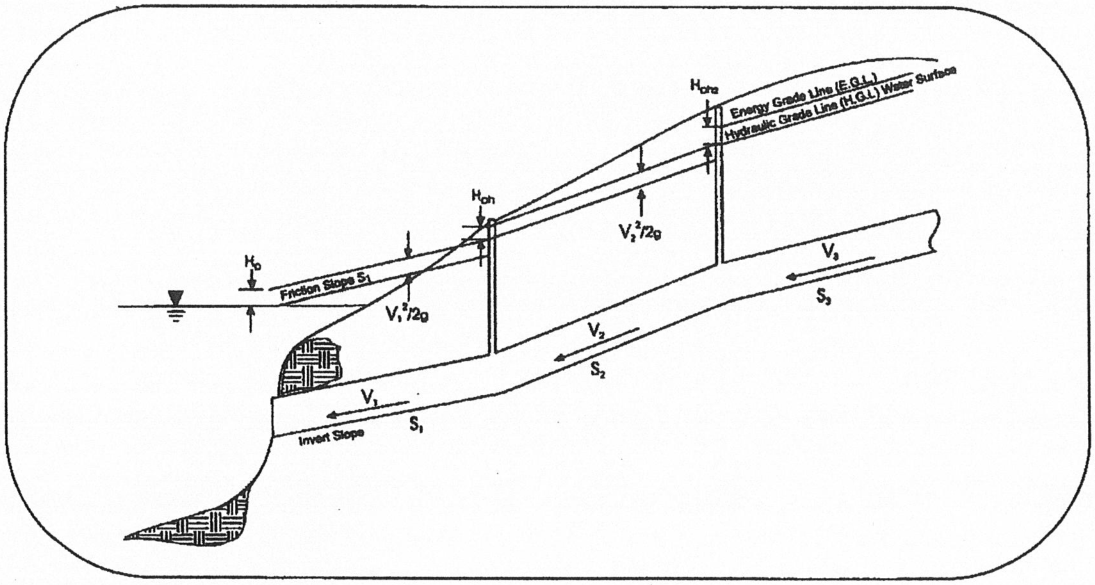
Publication No. FHWA-NHI-10-009
September 2009
(Revised August 2013)

U.S. Department
of Transportation

**Federal Highway
Administration**

Hydraulic Engineering Circular No. 22, Third Edition

URBAN DRAINAGE
DESIGN MANUAL



NATIONAL HIGHWAY INSTITUTE

Training Solutions for Transportation Excellence