EXAMPLE 11.11

Design a channel so that its maximum discharge is 2000 cfs. The alluvial material is fine sand. The available slope is 1 in 4000.

SOLUTION

1. $P_w = 2.67\ (2000)^{1/2} = 119$ ft
2. From Table 11.7, $c = 0.431$
   $R = cQ^{1/3} = 0.431(2000)^{1/3} = 5.42$ ft
3. $A = P_w R = 119(5.42) = 645\ \text{ft}^2$
4. Assuming side slopes of 1:1

$$A = (b + y)y = 645 \qquad \text{(a)}$$

and

$$P_w = (b + 2.83y) = 119 \qquad \text{(b)}$$

Solving (a) and (b)

$$b = 102 \text{ ft},\ y = 6 \text{ ft}$$

5. From Table 11.7, $s = 0.0009$
   $S = sQ^{-1/6} = 0.0009\ (2000)^{-1/6} = 0.00025$
   Available S is 1 in 4000 or 0.00025
   Computed slope is OK
6. Freeboard: From eq. (11.12), $u = \sqrt{(2.5)(6)} = 3.9$ ft
7. Total depth = 6 + 3.9 = 9.9 or 10 ft

11.12 GRADUALLY VARIED FLOW

11.12.1 Dynamic Equation of Gradually Varied Flow

When the gravity force causing the flow is not balanced with the resisting drag force, the depth varies gradually along the length of the channel. The dynamic equation of gradually varied flow is derived from the energy principle and indicates the slope of the water surface in the channel.

In Figure 11.11, the total energy at point 1, including the energy coefficient,* is

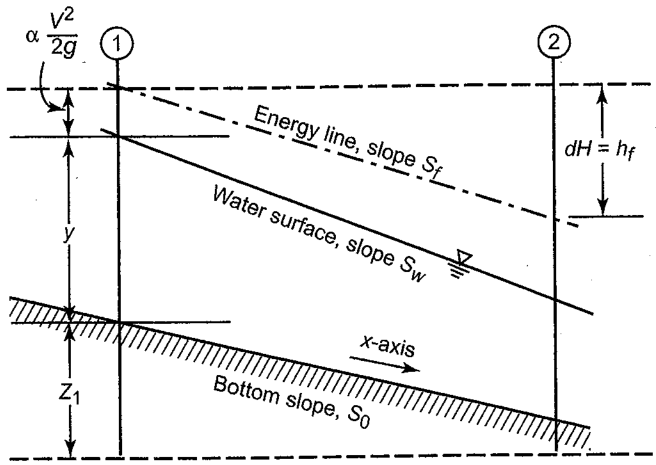
$$H = Z_1 + y + \alpha\frac{V^2}{2g} \qquad \text{(a)}$$

Differentiating with respect to the channel bottom as the x-axis:

$$\frac{dH}{dx} = \frac{dZ_1}{dx} + \frac{dy}{dx} + \alpha\frac{d}{dx}\left(\frac{V^2}{2g}\right) \qquad \text{(b)}$$

* The mean velocity is used in the relation. To account for the nonuniform distribution of velocity across a channel section, an energy coefficient α is included, expressed as $\alpha = \dfrac{\Sigma v^3 \Delta A}{V^3 A}$

Figure 11.11 Gradually varied flow.



If the level increasing in the direction of flow is assumed to be positive, then $dH/dx = -S_f$, $dZ_1/dx = -S_0$, and

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{V^2}{2g}\right)\frac{dy}{dx}$$

Equation (b) becomes

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + \alpha\left[d\left(V^2/2g\right)/dy\right]} \quad \text{[dimensionless]} \tag{11.36}$$

This is the equation of gradually varied flow. To reduce the equation further, it is considered that with the energy grade, S_f, used for the slope term in Manning's equation, that formula can be used for the gradually varied flow through a section, that is,

$$Q = \frac{1.49}{n} AR^{2/3} S_f^{1/2} \quad [\mathrm{L^3T^{-1}}] \quad \text{(English units)} \tag{11.37a}$$

$$Q = \frac{1}{n} AR^{2/3} S_f^{1/2} \quad [\mathrm{L^3T^{-1}}] \quad \text{(metric units)} \tag{11.37b}$$

or

$$Q = K\sqrt{S_f} \tag{c}$$

where K is the general expression for the conveyance. Also, in the case of uniform flow,

$$Q = K_n\sqrt{S_0} \tag{d}$$

where K_n is the normal flow conveyance. From eqs. (c) and (d),

$$\frac{S_f}{S_0} = \frac{K_n^2}{K^2} \tag{e}$$

The denominator term of eq. (11.36) may be developed as follows:

$$\alpha\frac{d}{dy}\left(V^2/2g\right)=\alpha\frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)=-\alpha\frac{Q^2}{g}\left(\frac{1}{A^3}\right)\frac{dA}{dy} \qquad \text{(f)}$$

Since $dA/dy = T$ and in general terms, $Z=\sqrt{A^3/T}$,

$$\alpha\frac{d}{dy}\left(\frac{V^2}{2g}\right)=-\alpha\frac{Q^2}{gZ^2} \qquad \text{(g)}$$

For the critical flow, $Z_c = Q/\sqrt{g/\alpha}$; hence

$$\alpha\frac{d}{dy}\left(\frac{V^2}{2g}\right)=-Z_c^2/Z^2 \qquad \text{(h)}$$

Substituting eqs. (e) and (h) in eq. (11.36), we have

$$\frac{dy}{dx}=S_0\frac{1-\left(K_n/K\right)^2}{1-\left(Z_c/Z\right)^2} \quad \text{[dimensionless]} \qquad (11.38)$$

Equation (11.38) is another form of the gradually varied flow equation, which is convenient for the evaluation.

11.12.2 Types of Flow Profile Curves

The integration of eq. (11.38) will represent the surface curve of the flow. The shape or profile of the surface curve depends on (1) the slope of the channel, and (2) the depth of flow compared to the critical and normal depths. The classification is as follows:

Sign convention

1. If the water surface is rising in the direction of flow, the curve, known as the *backwater curve*, is positive.
2. If the water surface is dropping, the curve, known as the *drawdown curve*, is negative.

Channel slopes

1. When $y_n > y_c$, the slope is mild.
2. When $y_n < y_c$, the slope is steep.
3. When $y_n = y_c$, the slope is critical.
4. When $S_0 = 0$, the slope is horizontal. For horizontal slope, $y_n = \infty$.
5. When $S_0 < 0$, the slope is adverse. For adverse slope, y_n is negative or nonexistent.

Flow profiles

If the lines are drawn at the critical depth and the normal depth parallel to the channel bottom, three zones are formed. Zone 1 is the space above the upper line, zone 2 is the space between the two lines, and zone 3 is the space between the lower line and the channel bottom. The zone in which the water surface lies determines the flow profile, as shown in Figure 11.12.

Figure 11.12 Types of flow profiles.

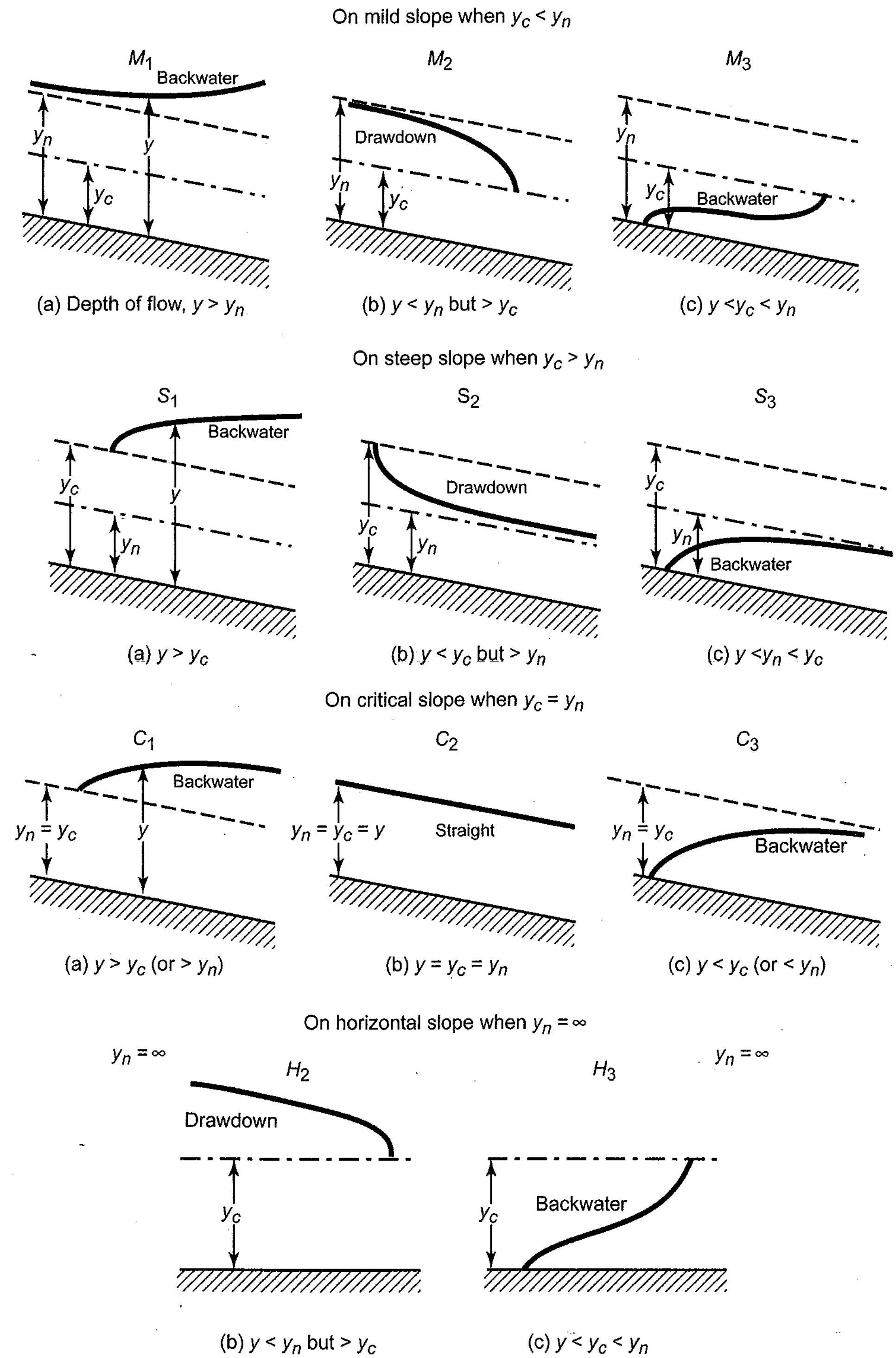
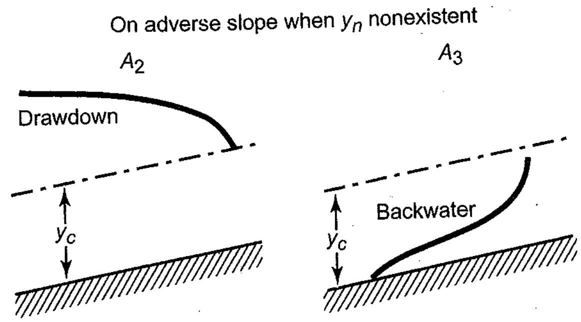


Figure 11.12 (Continued) Types of flow profiles.



(b) $y > y_c$ (y_n does not exist)

(c) $y < y_c$

11.12.3 Flow Profile Analysis

The analysis predicts the general shape of the flow profile in a longitudinal section of a channel without performing the quantitative analysis. For a channel of constant slope, the conditions described in the preceding section determine the type of flow profile. A break in the slope of a channel results in a change in the flow condition as well. A surface curve is often formed to negotiate the change of pattern of flow. Chow (1959, p. 232) has indicated 20 typical flow profiles for a combination of two different slopes by break in the channel slope. Certain points in the channel reach serve as a control section where the depth of flow is fixed (i.e., either it is y_c or y_n or has some other known value).

11.13 Computation of Flow Profile

The analytical determination of the shape of flow profile essentially is a solution of eq. (11.38). Since the variables on the right side of the equation cannot be expressed explicitly in terms of y, the exact integration of the equation is not practically possible. There are four approaches to computing the surface profile:

1. Graphical or numerical integration method
2. Analytical or direct integration method
3. Direct step method
4. Standard step method

Two of these are described in detail. The water surface elevation at the start of the curve from which the computation starts is a control section. The computation should proceed upstream from the control section in subcritical flow and in the downstream direction for supercritical flow.

11.13.1 Numerical Integration Method

Consider a channel section having a depth y_1 at x_1 and y_2 at x_2 as shown in Figure 11.13(a).

$$x_2 - x_1 = \int_{x_1}^{x_2} dx$$

or

$$x_2 - x_1 = \int_{y_1}^{y_2} \frac{dx}{dy} dy$$

The right side indicates the area under the dx/dy versus y curve, as shown in Figure 11.13(b). If a to b is considered a straight line for a small difference in y_1 and y_2, then

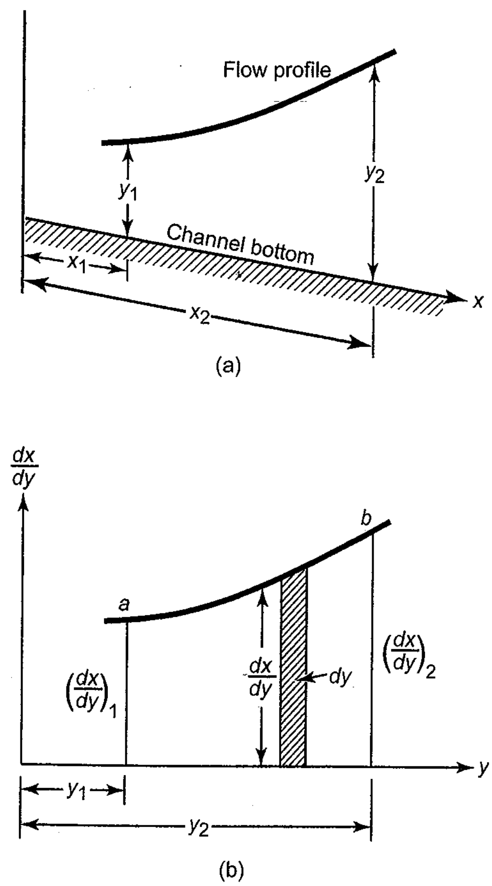
$$x_2 - x_1 = \frac{\left[(dx/dy)_1 + (dx/dy)_2\right]}{2}(y_2 - y_1) \quad [L] \tag{11.39}$$

The procedure comprises solving eq. (11.39) by the following steps:

1. Select several values of y starting from the control point.
2. For each value of y, calculate dx/dy by the inverse of eq. (11.38).
3. Using eq. (11.39), calculate x for two successive values of y.

For a backwater curve, the y values should be selected at close intervals near the tail part of the curve.

Figure 11.13 Derivation of numerical integration method.



EXAMPLE 11.12

A trapezoidal channel with a bottom width of 4 m and side slopes of 1:4 carries a discharge of 30 m^3/s. The channel has a constant bed slope of 0.001. A dam backs up the water to a depth of 3.0 m just behind the dam. Compute the backwater profile to a depth 5% greater than the normal channel depth. $n = 0.025$, $\alpha = 1.0$.

SOLUTION

1. The channel is the same as given in Examples 11.3 and 11.6.
2. From Example 11.3, critical depth $y_c = 1.22$ m
3. From Example 11.6, normal depth $y_n = 1.90$ m
4. Since $y_n > y_c$, the channel has a mild slope.
5. At the control point, y of 3 m is greater than y_n, thus the profile is of the M_1 type curve.
6. Section factor for critical flow,

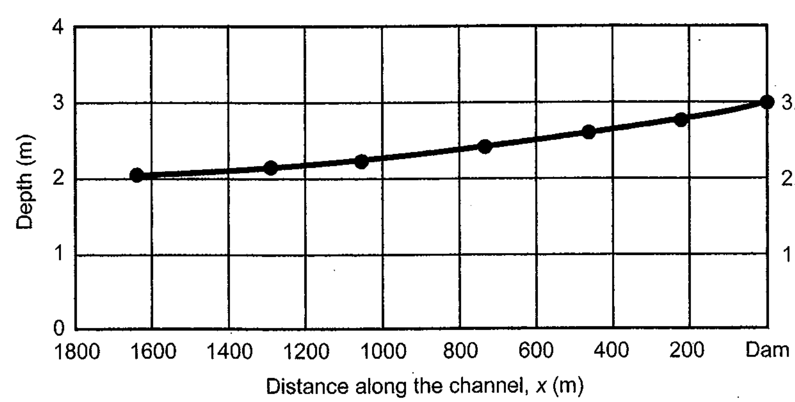
$$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{30}{\sqrt{9.81}} = 9.58$$

7. Conveyance for uniform flow,

$$K_n = \frac{Q}{\sqrt{S_0}} = \frac{30}{\sqrt{0.001}} = 948.68$$

8. At the starting point of the curve, the control section depth = 3 m. The last computed point which is 5% greater than 1.90 = 1.05(1.9) = 2.0 ft.
9. Since the flow is subcritical, computation proceeds upstream from the dam as the origin. The computations are arranged in Table 11.8.
10. The profile has been shown in Figure 11.14 by a plot between y (column 1) and x (column 10).

Figure 11.14 Backwater profile for Example 11.12.



11.13.2 Direct Step Method

This method directly uses the energy principle from section to section in the entire reach of the channel. It is applicable to prismatic channels. Applying the energy principle at points 1 and 2 of Figure 11.11 gives

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = Z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g} + h_f \tag{a}$$

or

$$(Z_1 - Z_2) - h_f = \left(y_2 + \alpha_2 \frac{V_2^2}{2g}\right) - \left(y_1 + \alpha_1 \frac{V_1^2}{2g}\right) \tag{b}$$

or

$$S_0 \Delta x - S_f \Delta x = E_2 - E_1 \tag{c}$$

where Δx is the distance between the two sections, E_1 and E_2 are specific energies, and $h_f = S_f \Delta x$, or

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \tag{d}$$

If the energy grade between the two sections is considered to be the average of the grade at sections 1 and 2, then

$$\Delta x = \frac{E_1 - E_2}{\bar{S}_f - S_0} \quad \text{[L]} \tag{11.40}$$

with

$$\bar{S}_f = \frac{S_{f1} + S_{f2}}{2} \quad \text{[dimensionless]} \tag{11.41}$$

and

$$S_f = \frac{V^2 n^2}{2.22 R^{4/3}} \quad \text{(English units)} \quad \text{[dimensionless]} \tag{11.42a}$$

$$S_f = \frac{V^2 n^2}{R^{4/3}} \quad \text{(metric units)} \quad \text{[dimensionless]} \tag{11.42b}$$

The steps of the procedure are as follows:

1. Select several values of y starting from the control point.
2. For a selected y, calculate A, R, $R^{4/3}$, and $V (= Q/A)$.
3. For a selected y, also calculate the velocity head $[\alpha(V^2/2g)]$, the specific energy, $E(= y + \alpha V^2/2g)$, and the energy slope, eq. (11.42).
4. For two successive values of y, determine the difference between the specific energy, ΔE, and the average of the energy slope, $\bar{S}_f$.
5. Compute Δx from eq. (11.40).

Table 11.8 Computation of the Flow Profile by the Numerical Integration Method

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
y Select:	T[a]	A[b]	R[c]	$R^{2/3}$	$K=\frac{1}{n}AR^{2/3}$[d]	$Z=\sqrt{\frac{A^3}{T}}$[d]	$\frac{dx}{dy}$ [Inverse of eq. (11.38)][e]	Δx [eq. (11.39)] (m)	x Cumulated (m)
3.0	28.0	48.0	1.67	1.41	2707.2	62.85	1113.50	228	228
2.8	26.4	42.56	1.57	1.35	2298.2	54.04	1167.50	243	471
2.6	24.8	37.44	1.47	1.29	1931.9	46.00	1260.60	270	741
2.4	23.2	32.64	1.37	1.23	1605.89	38.72	1442.03	333	1074
2.2	21.6	28.16	1.27	1.17	1317.89	32.15	1891.18	218	1292
2.1	20.8	26.04	1.22	1.14	1187.42	29.14	2466.0	332	1624
2.0	20.0	24.0	1.17	1.11	1065.60	26.29	4181.25		

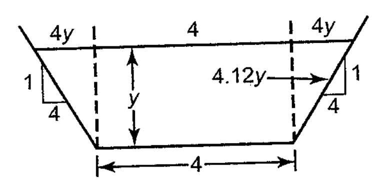
[a] $T = 4 + 8y$

[b] $A = (4 + 4y)y$

[c] $R = \frac{A}{P_w} = \frac{(4+4y)y}{4+8.24y}$

[d] K and Z calculated for each y selected.

[e] $\frac{dx}{dy} = \frac{1}{S_0}\frac{1-\left(\frac{Z_c}{Z}\right)^2}{1-\left(\frac{K_n}{K}\right)^2}$



The analytical integrating method requires use of the varied flow function tables (see Appendix D-2 of Chow, 1959). Many alternative computation procedures have been proposed under this method. The standard step method, also based on the energy principle, is a trial-and-error procedure wherein the depth of flow is determined for a given channel distance and not the inverse as in the other methods.

EXAMPLE 11.13

Determine the flow profile using the data of Example 11.12 by the direct step method.

SOLUTION The computations are arranged in Table 11.9.

11.14 RAPIDLY VARIED FLOW

This involves a sharp change in the curvature of the water surface, sometimes producing discontinuity in the flow profile. The streamlines are so disturbed that the pressure distribution is not hydrostatic. The rapid variation in flow conditions occurs within a short reach. As a result, the energy loss due to boundary friction is negligible with rapidly varied flow, while it is dominant in gradually varied flow conditions. The overall energy losses are substantial due to turbulent conditions. The problems related to rapidly varied flow are usually studied on an individual basis, with each phenomenon given a specific treatment. Flow over spillways and hydraulic jumps are two common cases of rapidly varied flow. The former is described in Chapter 9 and the latter is described below briefly.

11.14.1 Hydraulic Jump

When a shallow stream of high velocity impinges on water of sufficient depth, the result is usually an abrupt rise in the surface in the region of impact. This phenomenon is known as *hydraulic jump*. A similar phenomenon takes place when the flow passes from a steep slope to a mild slope or when an obstruction is met in the passage of a supercritical flow. For formation of a jump, the flow should be supercritical, which converts into subcritical flow after the jump.

In the process, a substantial loss of energy takes place. Since unknown energy losses are involved in the jump, the use of the energy principle is not practical. The principle of momentum is used instead, as described in Section 9.4. By this principle, the change in the forces between two sections is equated with the change in the rate of momentum, which is equal to the mass of water multiplied by the change in velocity. Commonly, the relation is developed for horizontal or slightly inclined channels in which the weight of water between the sections and the boundary friction are disregarded. It is applicable to most field channels.

The following formulas are derived for rectangular channels from the momentum principle on the basis described above:

$$\frac{D_2}{D_1} = \frac{1}{2}\left(\sqrt{1+8\text{Fr}_1^{\,2}} - 1\right) \quad \text{[dimensionless]} \tag{11.43}$$

where

D_1 and D_2 = depth before and after the jump

Fr_1 = Froude number before the jump = $V_1/\sqrt{gD_1}$

Table 11.9 Computation of the Flow Profile by the Direct Step Method

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
y Select:	A	$R=\frac{A}{P_w}$	$R^{4/3}$	$V=\frac{Q}{A}$	$\alpha\frac{V^2}{2g}$	$E=y+\frac{\alpha V^2}{2g}$	$\Delta E_s=E_1-E_2$	$S_f=\frac{n^2V^2}{R^{4/3}}$ $(\times 10^3)$	$\bar{S}_f$[a] $(\times 10^3)$	$\bar{S}_f-S_0$ $(\times 10^3)$	Δx [eq (11.40)]	x Cumulated
3.0	48.0	1.67	2.00	0.625	0.020	3.020		0.122				
							0.195		0.147	−0.853	228	228
2.8	42.56	1.57	1.82	0.705	0.025	2.825		0.171				
							0.192		0.207	−0.793	242	470
2.6	37.44	1.47	1.66	0.801	0.033	2.633		0.242				
							0.190		0.296	−0.704	270	740
2.4	32.64	1.37	1.51	0.919	0.043	2.443		0.350				
							0.185		0.433	−0.567	326	1066
2.2	28.16	1.27	1.37	1.065	0.058	2.258		0.517				
							0.090		0.578	−0.422	213	1279
2.1	26.04	1.22	1.30	1.152	0.068	2.168		0.638				
							0.088		0.716	−0.284	310	1589
2.0	24.0	1.17	1.23	1.250	0.080	2.080		0.794				

[a] Average of successive values of col. 9.

The depths D_1 and D_2 are referred to as conjugate depths. Equation (11.43) can be used to ascertain D_2 when D_1 is known. In the equation, subscripts 1 and 2 can be replaced by each other. Thus the equation can also be used to determine the prejump depth, D_1, for known post-jump depth, D_2 and Fr_2.

The discharge through the jump where b is the width of a rectangular channel can be given by

$$Q = b\left[(gD_1D_2)\frac{D_1+D_2}{2}\right]^{1/2} \quad [L^3T^{-1}] \tag{11.44}$$

The energy dissipated in a jump is computed from

$$E_{\text{loss}} = \frac{(D_2-D_1)^3}{4D_1D_2} \quad [L] \tag{11.45}$$

There are many applications of hydraulic jump. The main use is to dissipate energy in water flowing over spillways or weirs to prevent scouring downstream of the structure.

EXAMPLE 11.14

Water flows at a rate of 360 cfs in a rectangular channel of 18 ft width with a depth of 1 ft. **(a)** Is a hydraulic jump possible in the channel? **(b)** If so, what is the depth of flow after the jump? **(c)** How much energy is dissipated through the jump?

SOLUTION

(a) To determine the potential for hydraulic jump,

$$A = 18\times 1 = 18 \text{ ft}^2$$

$$V = \frac{Q}{A} = \frac{360}{18} = 20 \text{ ft/s}$$

$$Fr_1 = \frac{V_1}{\sqrt{gD_1}} = \frac{20}{\sqrt{32.2(1)}} = 3.52$$

Since $Fr_1 > 1$, supercritical flow, jump can form.

(b) Depth of flow post-jump,

$$\frac{D_2}{D_1} = \frac{1}{2}\left(\sqrt{1+8(3.52)^2}-1\right) = \frac{9.0}{2}$$

$$D_2 = \frac{1}{2}(9.0)(1) = 4.50 \text{ ft}$$

(c) Loss of energy,

$$E_{\text{loss}} = \frac{(D_2-D_1)^3}{4D_1D_2}$$

$$= \frac{(4.5-1.0)^3}{4(1.0)(4.5)}$$

$$= 2.38 \text{ ft-lb/lb}$$

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