Solution From eq. (3.12),

$$b' = (150 \text{ cm})\left[\frac{1}{100}\frac{\text{cm}}{\text{m}}\right] = 1.5 \text{ m}$$

$$L_e = \frac{0.008}{1.5} = 0.0053 \text{ per day}$$

From eq. (3.13),

$$a = \frac{2.5}{0.0053} = 471.7 \text{ m}$$

From eq. (3.14),

$$B = \sqrt{\frac{2.5 \times 20}{0.0053}} = 97.1 \text{ m}$$

3.7 Parameters of Groundwater Storage

Two important aspects of the study of groundwater are the movement of water underground to streams and wells and underground storage in which an aquifer serves as a storage reservoir. The volume of water taken or released from storage with changes in water levels is reflected in the parameters of *specific yield* or specific retention for water-table aquifers and by *specific storage* or storage coefficient for confined aquifers. The fundamental parameter of groundwater phenomena, however, is porosity, which allows soil to be considered a porous medium.

3.7.1 Porosity

An element of soil, separated in three phases, is shown schematically in Figure 3.10. Porosity is defined as the ratio of the volume of voids to the total volume, or

$$\eta = \frac{V_v}{V_t} \quad \text{[dimensionless]} \tag{3.15}$$

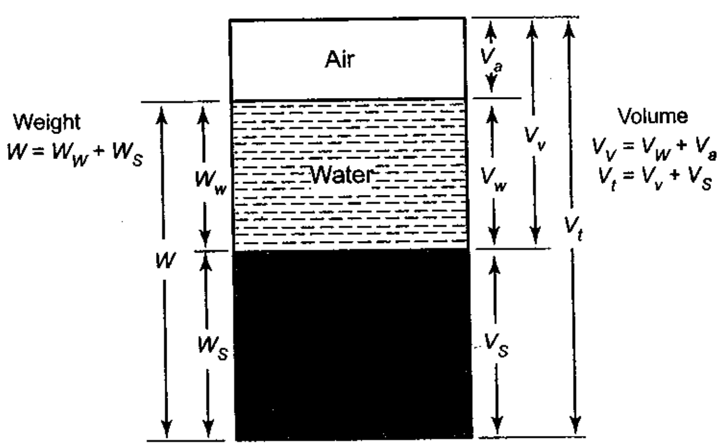


Figure 3.10 Three phases in a soil element.

The term *void ratio* is commonly used in soil mechanics to provide an indication of voids or pores in the soil. It is defined as the ratio of the volume of voids to the volume of solids in a soil sample, or

$$e = \frac{V_v}{V_s} \quad \text{[dimensionless]} \tag{3.16}$$

This term, however, is rarely used in groundwater flow.

Porosity and void ratio are interrelated by the expression

$$e = \frac{\eta}{1-\eta} \quad \text{[dimensionless]} \tag{3.17}$$

Bulk (dry) density, ρ_b, of soil is the mass of soil solids (dry soil) per unit gross volume of soil, and the density of soil particles (grains), ρ_s is equal to the mass of soil solids per unit volume of soil solids. For the same mass of soil solids,

$$\rho_b \propto \frac{1}{V_t} \text{ and } \rho_s \propto \frac{1}{V_s}$$

where

ρ_b = dry (bulk) density

V_t = total soil volume

ρ_s = grain density

V_s = dry soil volume

From these relations and eq. (3.15), the following relation emerges:

$$\frac{\rho_b}{\rho_s} = 1 - \eta$$

or

$$\frac{G_b}{G_s} = 1 - \eta \quad \text{[dimensionless]} \tag{3.18}$$

where G_b and G_s are bulk specific gravity and specific gravity of soil solids, respectively.

Porosity is a measure of the water-bearing capacity of a formation. However, it is not just the total magnitude of porosity that is important from the consideration of water extraction and transmission, but the size of voids and the extent to which they are interconnected since pores may be open (interconnected) or closed (isolated). For instance, a clay formation may have a very high porosity but it is a poor medium as an aquifer. Specific yield, or effective porosity, and specific retention, as discussed below, are important from this consideration.

EXAMPLE 3.10

A sample of sandy soil is collected from an aquifer. The sampler with a volume of 50 cm^3 is filled with the soil. When the soil is poured into a graduated cylinder, it displaces 30.5 cm^3 of water. What are the porosity and the void ratio of the sand?

SOLUTION

The volume of water displaced is equal to the volume of soil particles (solids); thus $V_s = 30.5$ cm^3 while $V_t = 50$ cm^3
Hence

$$V_v = V_t - V_s = 50 - 30.5 = 19.5 \text{ cm}^3$$

From eq. (3.15),

$$\eta = \frac{V_v}{V_t} = \frac{19.5}{50.0} = 0.39 \text{ or } 39\%$$

From eq. (3.17),

$$e = \frac{\eta}{1-\eta} = \frac{0.39}{1-0.39} = 0.64$$

EXAMPLE 3.11

A soil sample occupies 0.132 ft^3. When dried, it weighs 15.8 lb. If the specific gravity of soil solids is 2.65, calculate **(a)** the bulk density of the soil, and **(b)** the porosity of the soil.

SOLUTION

1. Dry unit weight $= \dfrac{15.8}{0.132} = 119.7 \text{ lb/ft}^3$

2. Dry (bulk) density, $\rho_b = \dfrac{119.7}{g} = \dfrac{119.7}{32.2} = 3.72 \text{ slugs/ft}^3$

3. Unit weight of soil grains $= G_s \gamma_w = (2.65)(62.4)$
$$= 165.36 \text{ lb/ft}^3$$

4. Density of soil grains, $\rho_s = \dfrac{165.36}{g} = \dfrac{165.36}{32.2} = 5.14 \text{ slugs/ft}^3$

5. From eq. (3.18),
$$\eta = 1 - \frac{3.72}{5.14} = 0.276 \text{ or } 27.6\%$$

6. [Instead of dry weight, the natural (wet) weight of 18 lb/ft^3 with a moisture content, ω, of 14% could have been given in the problem. In such a case,
$$W_s = \frac{W_t}{1+\omega/100} = \frac{18}{1+14/100} = 15.8 \text{ lb}$$
Other steps are the same as above.]

3.7.2 Specific Retention (of Water-Table Aquifer)

When the water table is lowered, water drains from the pore spaces of an aquifer and is replaced with air. This process occurs because the pressure of water inside the pores becomes less than the surrounding air pressure. However, a part of the water is retained within the pores, due to forces of *adhesion* (attraction between pore walls and adjacent water molecules) and *cohesion* (attraction between molecules of water), which are stronger than the pressure difference between the air pressure and the water pressure. The difference of air pressure and water pressure is known as *capillary pressure*, P_c. The volume of water thus retained against the force of gravity, compared to the total volume of rock (soil), is called the *specific retention*. It is also known as the *field capacity* or water-holding capacity. This is a measure of the water-retaining capacity of the porous medium. Specific retention is thus dependent on both pore characteristics and factors affecting the surface tension, such as temperature, viscosity, mineral composition of water, and so on.

As stated above, the amount of water drained from the saturated soil is a function of capillary pressure. A characteristic curve of this function is shown in Figure 3.11. As P_c increases, the volumetric-moisture content* decreases. At a large value of P_c, the volumetric-moisture content tends toward a constant value because of adhesion and cohesion (explained earlier) and the gradient $\Delta\omega/\Delta P_c$ approaches zero. The volumetric-moisture content at this state is equal to the specific retention, as shown in Figure 3.11.

A simple device consisting of a porous plate, capillary tube, and leveling bottles is used to measure volumetric-moisture content and capillary pressure head on a saturated sample. The data are plotted as in Figure 3.11 to obtain the specific retention of the representative sample.

The porewater pressure at any depth h below the water table is equal to γh like hydrostatic pressure, or simply h in terms of water head. Thus, pressure above the water table, with reference to the water table as a datum, will be negative and equal to the height of the point from the water table. This negative pressure is simply the capillary pressure, P_c. If we follow the relationship of Figure 3.11 between capillary pressure and moisture content, the same curve indicates moisture content (in volumetric terms) of the soil at various heights above the water table. Consider the water table in Figure 3.12a at level 1; the moisture distribution curve will be as shown by the outer solid curve in Figure 3.12b. Suppose that the water table drops down to level 2. When equilibrium is achieved, the moisture distribution curve will be similar to level 1 but will be displaced to level 2, as shown by the dotted curve in Figure 3.12b. The area under the curve represents the moisture in the soil. The shaded area between the two curves or at the base between the two water-table lines represents the amount of water drained from the soil with the reduction of water table from level 1 to level 2.

EXAMPLE 3.12

A 200-g dry soil sample is tested by a porous plate test. The negative (capillary) pressure head and the incremental amount of water released from the sample are indicated below. The bulk density of the soil is 1.5 g/cc and at saturation the weight moisture content of the soil is 29.33%. **(a)** Calculate the volumetric-moisture content of the soil for each capillary pressure

* Moisture content, ω, is a weight parameter. Here this term is used to indicate the quantity of water inside the pores in terms of volume. The following relation holds:

$$\text{volumetric moisture content} = \text{weight moisture content} \times \frac{\text{bulk density of soil}}{\text{density of water}}$$

or volumetric moisture content = weight moisture content × bulk specific gravity

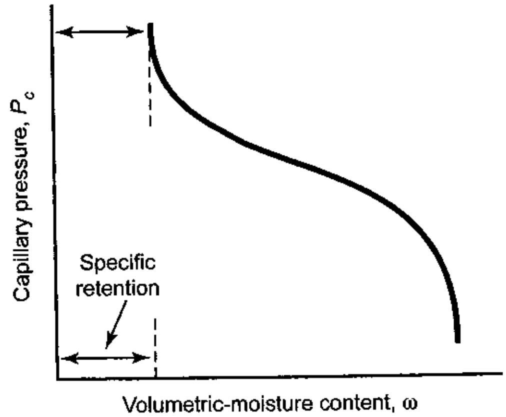
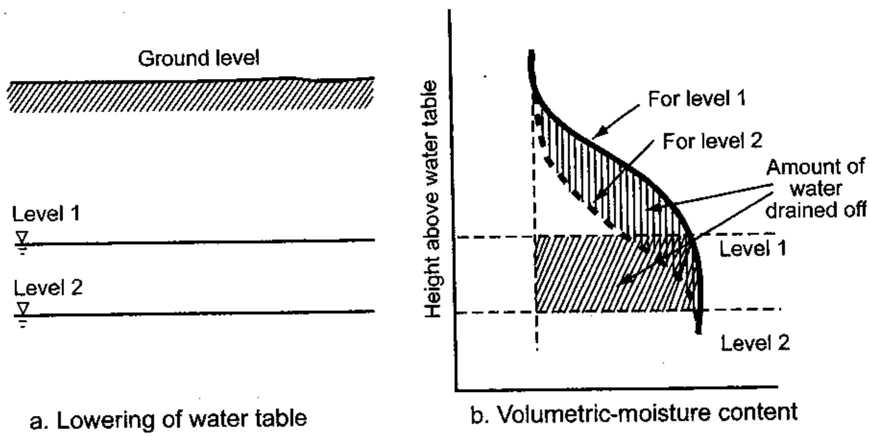


Figure 3.11 Soil-water retention curve.

Figure 3.12 Water drained with lowering of the water table.



head and plot the moisture distribution curve. **(b)** If the water table was initially located 300 cm below the surface and subsequently receded to 350 cm below the surface, calculate the volume of water removed from the soil per unit area and the specific retention for the soil.

Capillary head, cm	0	20	50	80	100	120	150	170	200	230	250	280	300
Incremental water release, cm^3	0	0	1.34	3.33	3.34	4.00	6.66	4.00	6.67	5.33	3.33	2.00	0

SOLUTION

1. Volume of soil sample $= \dfrac{\text{dry mass}}{\text{bulk density}} = \dfrac{(200\text{ g})}{(1.5\text{g/cc})} = 133.33\text{ cm}^3$

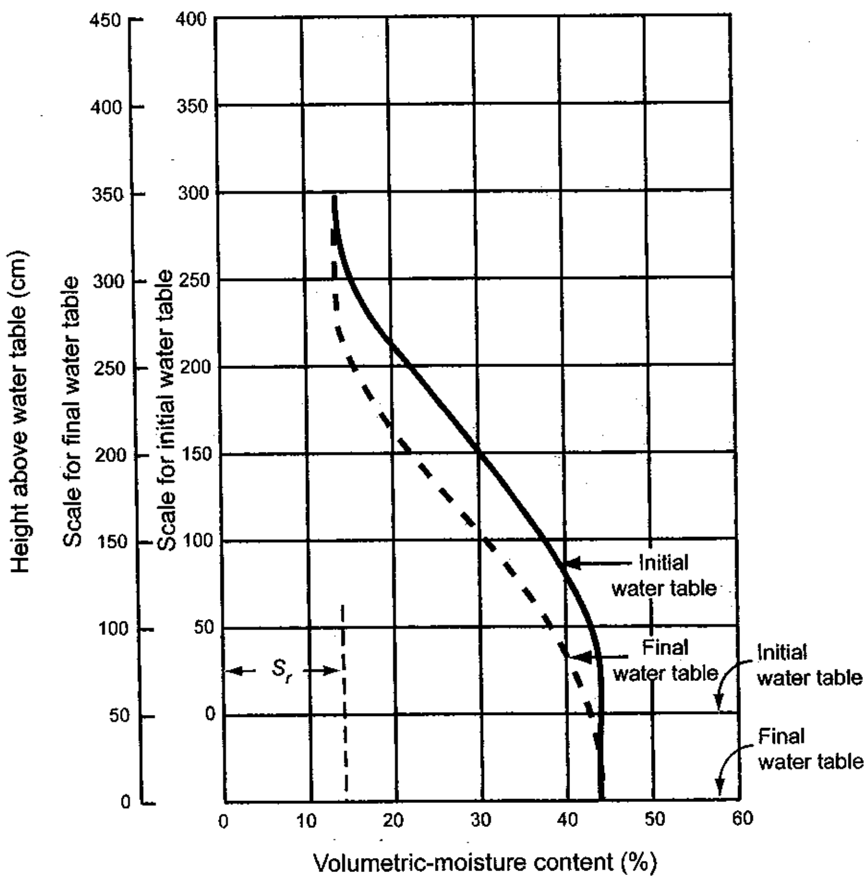
2. Volumetric moisture content (at saturation)

$$= (\text{weight moisture content})\frac{(\text{soil bulk density})}{(\text{water density})}$$

$$= (29.33)\left(\frac{1.5}{1}\right) = 44\% \quad \text{or} \quad 0.44$$

3. Volume of water in sample (at saturation) = 0.44(133.33) = 58.67 cm^3
4. Volume of water retained in the soil, as indicated in col. 4 of Table 3.4, equals the volume of water at saturation minus the total water released of col. 3.
5. Volumetric moisture content at various capillary heads in col. 5 is the water retained in col. 4 divided by the volume of soil.
6. The values of capillary head and moisture content are plotted on a graph designated initial water table, as shown by the solid curve in Figure 3.13.

Figure 3.13 Amount of water drained with lowering of water level in Example 3.12.



7. The scale on the y-axis is scaled down by 50 cm to represent the lowering of the water table. The values of capillary head and moisture content are now plotted on this revised scale as shown by the dotted curve on the figure. The volume of water removed per unit soil area is represented by the area between the two curves.
8. The area between the two curves = 1500 cm-%.
9. The volume of water removed $= \frac{1500}{100^*} = 15 \text{ cm}^3$ per cm^2 area
10. The total volume of soil per unit surface area between the two water tables = 1 × 50 = 50 cm^3.
11. The water removed per unit (1 cm^3) volume of soil = 15/50 = 0.30 or 30%
12. The specific retention, S_r, = 14% (from the figure).

Table 3.4 Moisture Contents for Various Capillary Heads

(1) Capillary head, cm	(2) Incremental water released, cm^3	(3) Total water released, cm^3	(4) Water retained, cm^3	(5) Volumetric-moisture content, %
0	0	0	58.67	44
20	0	0	58.67	44
50	1.34	1.34	57.33	43
80	3.33	4.67	54.00	40.5
100	3.34	8.01	50.66	38
120	4.00	12.01	46.66	35
150	6.66	18.67	40.00	30
170	4.00	22.67	36.00	27
200	6.67	29.34	29.33	22
230	5.33	34.67	24.00	18
250	3.33	38.00	20.67	15.5
280	2.00	40.00	18.67	14
300	0	40.00	18.67	14

3.7.3 Specific Yield (of Water-Table Aquifer)

Specific yield, also known as *effective porosity*, is defined as the volume of water yielded by an unconfined aquifer by gravity as compared to the unit volume of the aquifer. As the water level falls, water is drained from the pores. Specific yield can not be determined for a confined aquifer since the aquifer is not drained. Specific yield is given by

$$S_y = \frac{\text{Volume of water yielded by gravity}}{\text{Volume of unconfined aquifer}} \times 100$$

* Since the moisture content is in %.

or

$$S_y = \frac{1}{A}\frac{dV}{dh} \quad \text{[dimensionless]} \tag{3.19}$$

where

S_y = specific yield

A = area of soil formation

dV = volume of water drained

dh = change in water table

Since some water remains in the soil, the sum of specific yield and specific retention is equal to the porosity, or

$$S_y = \eta - S_r \quad \text{[dimensionless]} \tag{3.20}$$

where

S_y = specific yield

S_r = specific retention

In addition to the capillary head moisture content technique discussed in the preceding section, there are other procedures for determining specific yield, including the well-pumping tests discussed in Chapter 4.

Table 3.5 indicates the representative values of specific yield for various types of soils and rocks. The specific yield of most aquifer formations ranges from about 0.10 to about 0.30 and averages 0.20.

Table 3.5 Representative Values of Specific Yield for Soils and Rocks

Formation	Range of values	Typical
Gravel, coarse	0.10–0.25	0.21
Gravel, medium	0.15–0.45	0.24
Gravel, fine	0.15–0.40	0.28
Sand, coarse	0.15–0.45	0.30
Sand, medium	0.15–0.45	0.32
Sand, fine	0.01–0.45	0.23
Silt	0.01–0.40	0.20
Till, gravel	0.05–0.20	0.16
Till, sand		0.16
Till, silt		0.06
Clay	0.01–0.20	0.06
Sandstone, medium grained	0.01–0.40	0.27
Sandstone, fine grained		0.21
Limestone	0.01–0.35	0.14
Siltstone	0.01–0.35	0.12

EXAMPLE 3.13

For Example 3.12, determine the porosity and specific yield of the soil.

SOLUTION

Porosity = volumetric-moisture content at saturation or zero capillary head

$$\eta = 44\% \text{ (from Figure 3.13)}$$

From eq. (3.20), $S_y = 44 - 14 = 30\%$
(Note that the water drained out in Example 3.12 is equal to the specific yield.)

EXAMPLE 3.14

A water table drops 5 ft over an area of 3.5 acres. If the soil has a specific yield of 4%, how much water has drained from the area?

SOLUTION

1. The area of 3.5 acres = $3.5 \times 43{,}560 = 152{,}460 \text{ ft}^2$
2. The total volume of soil drained off = $5 \times 152{,}460 = 762{,}300 \text{ ft}^3$
3. The volume of water = $S_y \times$ total volume of soil drained off (by definition)

$$= 0.04 \times 762{,}300$$

$$= 30{,}492 \text{ ft}^3$$

In water-table aquifers, some quantity is derived from compression of the aquifer and change of density of the water. Therefore the storage coefficient for water-table aquifers is the total specific yield plus the fraction attributable to compressibility. The latter is, however, negligible compared to gravity drainage; specific yield provides an indication of aquifer release. The term storage coefficient is generally used in relation to confined aquifers.

3.7.4 Specific Storage for Confined Aquifers

The term *specific storage* is defined as the volume of water released from storage per unit decline in pressure head within the unit volume of an aquifer. It is a constant property of an aquifer and, as such, is a more fundamental parameter. A confined aquifer remains saturated at all times and, as such, water release is not derived from drainage of the voids by gravity as in the case of unconfined aquifers. In confined aquifers, the release or addition of water is attained due to the change in pore pressure.

In an equilibrium condition, the forces due to the weight of the formations overlying the aquifer and all other loads from the top are balanced by the skeleton and water within the pores of the aquifer. Due to the pumping of a well, the water pressure inside the pores is reduced. This results in a slight compaction of the skeleton of the aquifer and expansion of the water permitted by its elasticity. A certain amount of water is thus released from storage. The reverse process takes place in response to recharge.

Jacob made the first attempt in 1940 to introduce an analytical expression for the *specific storage.* For an elastic confined aquifer, he defined

$$S_s = \eta\gamma_w\left(\frac{1}{\eta E_s}+\frac{1}{E_w}\right) \quad [L^{-1}] \qquad (3.21a)$$

or

$$S_s = \rho g(\alpha+\eta\beta) \quad [L^{-1}] \qquad (3.21b)$$

where

S_s = specific storage

E_w = bulk modulus of elasticity of water (3×10^5 psi at ordinary temperatures)

E_s = bulk modulus of elasticity of soil solids

α = aquifer compressibility $(1/E_s)$

β = water compressibility $(1/E_w)$

η = porosity

The first term of the expression in parentheses relates to the compressibility of the aquifer and the second term to the expansibility of water.

DeWiest (1966) criticized this derivation, which had considered deformation of the aquifer (one side of the volume element was considered deformable). Cooper (1966) made a further refinement considering flow rate relative to moving grains of the aquifer medium and the flow rate across the fixed boundaries of the control volume. For a very small grain velocity, his form reduces to Jacob's formulation. Table 3.6 indicates the range of values of specific storage of soils and rocks.

Table 3.6 Specific Storage Values

Formation	Specific storage, m^{-1}
Gravel, dense sandy	$1.0 \times 10^{-4} - 4.9 \times 10^{-5}$
Sand, dense	$2.0 \times 10^{-4} - 1.3 \times 10^{-4}$
Sand, loose	$1.0 \times 10^{-3} - 4.9 \times 10^{-4}$
Clay, medium hard	$1.3 \times 10^{-3} - 9.2 \times 10^{-4}$
Clay, stiff	$2.6 \times 10^{-3} - 1.3 \times 10^{-3}$
Clay, plastic	$2.0 \times 10^{-2} - 2.6 \times 10^{-3}$
Rock, fissured	$6.9 \times 10^{-5} - 3.3 \times 10^{-6}$
Rock, unfissured	$< 3.3 \times 10^{-6}$

3.7.5 Storage Coefficient or Storativity

The *storage coefficient* is the volume of water that is released or taken into storage by an aquifer per unit area of the aquifer per unit decline or rise in pressure head. Storage coefficient is expressed as

$$S = \frac{S_y}{100} + S_s b \quad [\text{dimensionless}] \qquad (3.22)$$

In a confined aquifer, S_y is zero. Thus for a confined aquifer the relation is

$$S = S_s b \quad \text{[dimensionless]} \tag{3.23}$$

In contrast to the specific yield of an unconfined aquifer, the storage coefficient of the confined aquifer is much smaller, ranging from about 10^{-5} to 10^{-3}.

In an unconfined aquifer, S_s has no relevance. Hence, storage coefficient is similar to the specific yield in decimal points.

EXAMPLE 3.15

A confined aquifer of 40 m thickness has a porosity of 0.3. Determine the specific storage and storage coefficient. $\alpha = 1.5 \times 10^{-9}$ cm²/dyn, $\beta = 5 \times 10^{-10}$cm²/dyn.

SOLUTION

1. $\rho g = (1 \text{ g/cm}^3)(980 \text{ cm/s}^2) = 980 \text{ dyn/cm}^3$
   $b = (40 \text{ m})[100 \text{ cm/m}] = 4000 \text{ cm}$
2. Specific storage [from eq. (3.21b)]:

$$\begin{aligned} S_s &= \gamma_w(\alpha + \eta\beta) \\ &= \left(980\frac{\text{dyn}}{\text{cm}^3}\right)\left(1.5\times10^{-9} + 0.3\times5\times10^{-10}\frac{\text{cm}^2}{\text{dyn}}\right) \\ &= 1.62\times10^{-6} \text{ per cm} \end{aligned}$$

3. Storage coefficient [from eq. (3.22)]:

$$\begin{aligned} S &= 1.62\times10^{-6}\times4000 \\ &= 6.47\times10^{-3} \end{aligned}$$

EXAMPLE 3.16

The storage coefficient determined from a pumping test in an aquifer is 4×10^{-4} at a location where the aquifer depth is 100 ft. If the average volume of the aquifer per square foot is 80 ft³, how much water will be released by the aquifer with a drop in head of 70 ft?

SOLUTION

1. $S_s = \dfrac{4\times10^{-4}}{100} = 4\times10^{-6}$ per foot per unit volume
2. The amount of water released is

$$(4 \times 10^{-6})(80)(70) = 0.022 \text{ ft}^3 \text{ per ft}^2 \text{ of area}$$

3. Note that if the storage coefficient is used directly, the volume of water released is

$$(4 \times 10^{-4})(1 \text{ ft}^2)(70) = 0.028 \text{ ft}^3 \text{ per ft}^2$$

This is incorrect because the average thickness of the aquifer is 70 ft, whereas the storage coefficient is based on a 100-ft depth.

4. To use the storage coefficient, the following procedure has to be followed:

$$S_s = 4 \times 10^{-6} \text{ per foot}$$
$$S \text{ for aquifer} = (4 \times 10^{-6})(70) = 2.8 \times 10^{-4}$$

The amount of water released is

$$(2.8 \times 10^{-4})(1)(80) = 0.022 \text{ ft}^3 \text{ per ft}^2$$

3.8 Generalization of Darcy's Law

Darcy's law has been presented for one-dimensional flow in Section 3.5, which is the form in which it was empirically proposed by Darcy. However, at any point of a fluid in three-dimensional flow, there are three velocity components, a pressure component, and a density component. In groundwater flow, density is commonly considered constant (water is assumed to be incompressible unless specifically stated to the contrary, as in storage coefficient computation for an artesian aquifer). When water flows through an inclined medium as shown in Figure 3.3, its pressure (piezometric) head, which is a scalar quantity, is written as

$$h = Z + \frac{p}{\gamma} \quad [\text{L}] \tag{3.24}$$

There also is a kinetic energy term, which can be disregarded in considering the loss of head due to flow.

The velocity (specific discharge) component, however, is a vector quantity and can be expressed in three directions by Darcy's equation (3.4), with a negative sign for differential form of downward gradient:

$$\begin{aligned} v_x &= -K\frac{\partial h}{\partial x} \quad [\text{LT}^{-1}] \\ v_y &= -K\frac{\partial h}{\partial y} \\ v_z &= -K\frac{\partial h}{\partial z} \end{aligned} \tag{3.25}$$

If **i**, **j**, **k** represent standard unit vectors in the x, y, and z directions, respectively, the velocity (specific discharge) components in the three coordinate directions will be $\mathbf{i}v_x$, $\mathbf{j}v_y$, and $\mathbf{k}v_z$. The resultant velocity (specific discharge) vector will be given by

$$v = \mathbf{i}v_x + \mathbf{j}v_y + \mathbf{k}v_z$$

Treating K as constant and substituting eq. (3.25), we have

$$v = -K\left\{\mathbf{i}\frac{\partial h}{\partial x} + \mathbf{j}\frac{\partial h}{\partial y} + \mathbf{k}\frac{\partial h}{\partial z}\right\}$$

or

$$v = -K\nabla h \quad [\text{LT}^{-1}] \tag{3.26}$$

where ∇h denotes the head-gradient vector.

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