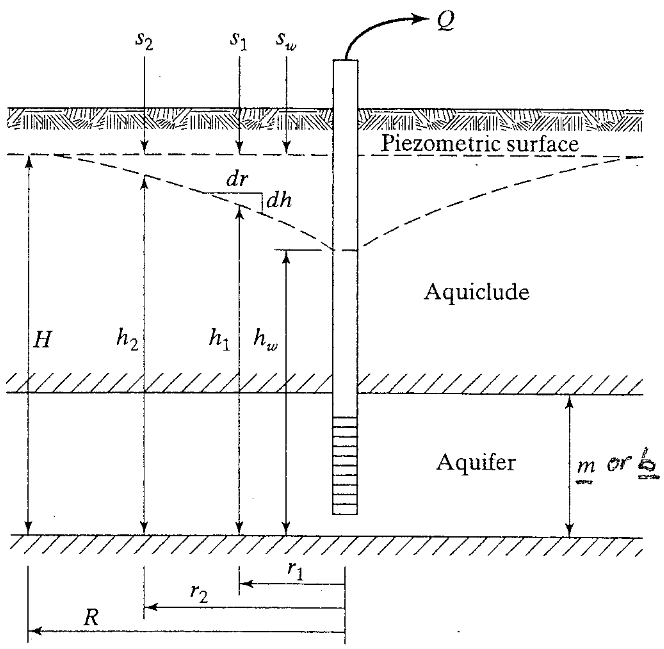
Steady-State Groundwater Flow:

Single Well Pumping in Confined Aquifer – Thiem's Equation



$$Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)}$$

$$s_w = H - h_w = \frac{Q}{2\pi T}\ln(R/r_w)$$

Example 9.7

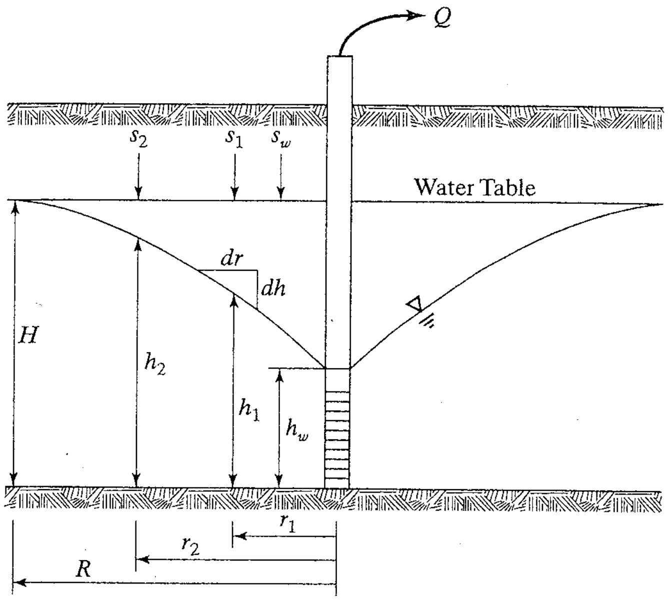
Steady-State Drawdown in a Confined Aquifer

A 24-inch diameter well is installed in a confined aquifer using a 6-inch gravel pack ($r_w = 1.5$ ft). The aquifer has a transmissivity of 40,000 gpd/ft, and the well is pumped at a rate of 2,000 gpm. If the radius of influence (R) of the well is 100,000 ft, determine the steady-state drawdown at the well.

$$s_w = \frac{Q}{2\pi T}\ln\left(\frac{R}{r_w}\right) = \frac{2{,}000 \times 1{,}440}{2\pi \times 40{,}000}\ln\left(\frac{100{,}000}{1.5}\right) = 127 \text{ ft}$$

Steady-State Groundwater Flow:

Single Well Pumping in Unconfined Aquifer – Dupuit's Equation



$$Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

Example 9.8

Steady-State Drawdown in an Unconfined Aquifer

A 12-inch diameter well is installed in an unconfined aquifer with a saturated thickness of 100 ft. The aquifer has a transmissivity of 40,000 gpd/ft, and the well is pumped at a rate of 600 gpm. If the radius of influence (R) of the well is 4,000 ft, determine the steady-state drawdown at the well.

$$h_2^2 - h_1^2 = \frac{Q}{\pi K}\ln\left(\frac{r_2}{r_1}\right)$$

$$100^2 - h_w^2 = \frac{600 \times 1{,}440}{3.1416 \times 400}\ln\left(\frac{4{,}000}{0.5}\right)$$

$$h_w^2 = 10{,}000 - 6{,}179$$

$$h_2 = 61.8 \text{ ft}$$

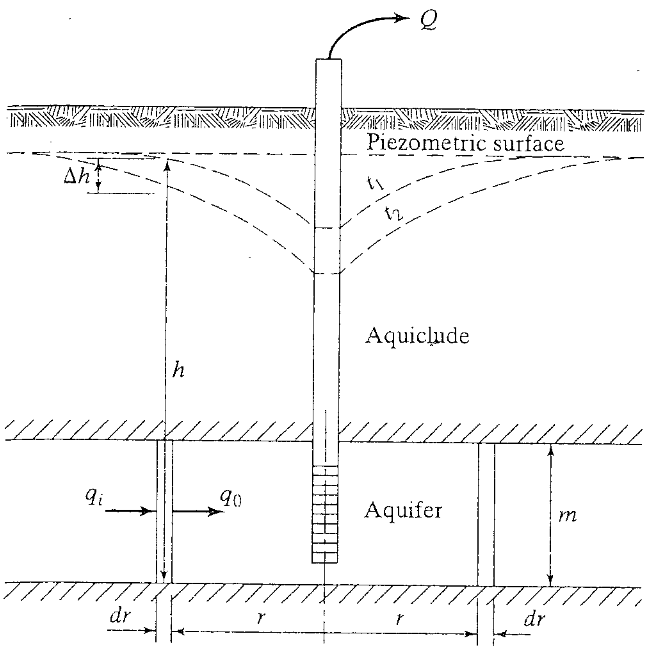
$$s_w = h_2 - h_w = 38.2 \text{ ft}$$

For comparison, the computed drawdown using the confined aquifer equation is

$$s_w = \frac{Q}{2\pi T}\ln\left(\frac{R}{r_w}\right) = \frac{600 \times 1{,}440}{2\pi \times 40{,}000}\ln\left(\frac{4{,}000}{0.5}\right) = 31.0 \text{ ft}$$

Unsteady-State Groundwater Flow:

Single Well Pumping in Confined Aquifer – Theis's Equation



$$s = \frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-u}}{u}\,du = \frac{Q}{4\pi T}W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

$$W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \cdots$$

Example 9.9

Unsteady-State Drawdown in a Confined Aquifer

For the well in Example 9.7, determine the drawdown at the well ($r_w = 1.5$ ft) and at $r = 100{,}000$ ft after 1 year of pumping at a rate of 2,000 gpm. The storage coefficient of the aquifer is 0.00035.

$$s = \frac{Q}{4\pi T}W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

Drawdown at the well

$$u = \frac{1.5^2(0.00035)7.48}{4 \times 40{,}000 \times 365} = 1.01 \times 10^{-10}$$

$$W(u) = -0.5772 - \ln u + u = -0.5772 + 23.02 + 0.0 = 22.44$$

$$s_w = \frac{2{,}000 \times 1{,}440}{4\pi\ 40{,}000} \times 22.44 = 128.6 \text{ ft}$$

Drawdown at $r = 100{,}000$ ft

$$u = \frac{(100{,}000)^2(0.00035)7.48}{4 \times 40{,}000 \times 365} = 0.45$$

$$W(u) = -0.577 - \ln(0.45) + 0.45 - \frac{0.45^2}{4} = 0.62$$

$$s = \frac{2{,}000 \times 1{,}440}{4\pi\ 40{,}000} \times 0.62 = 3.6 \text{ ft}$$

Unsteady-State Groundwater Flow:

Single Well Pumping in Confined Aquifer – Theis's Equation *(Cont.)*

Example 9.10 Unsteady-State Drawdown in an Unconfined Aquifer

(a) The unconfined aquifer in Example 9.8 has a storage coefficient of 0.15. Compute the drawdown for r = 0.5 ft and 4,000 ft after 1 year of pumping at a rate of 600 gpm using the Thesis equation

$$s = \frac{Q}{4\pi T} W(u)$$

Drawdown for r = 0.5 ft

$$u = \frac{r^2 S}{4Tt} = \frac{0.5^2 \times 0.15 \times 7.48}{4 \times 40{,}000 \times 365} = 4.8 \times 10^{-9}$$

$$W(u) = -0.577 - \ln(u) + u = -0.577 - \ln(4.8 \times 10^{-9}) + 0 = 18.6$$

$$s = \frac{600 \times 1{,}440}{4\pi\ 40{,}000} \times 18.6 = 31.9 \text{ ft}$$

Drawdown at r = 4,000 ft

$$u = \frac{4{,}000^2 \times 0.15 \times 7.48}{4 \times 40{,}000 \times 365} = 0.307$$

$$W(u) = -0.577 - \ln(0.307) + 0.307 - \frac{(0.307)^2}{4} = 0.88$$

$$s = \frac{600 \times 1{,}440}{4\pi\ 40{,}000} \times 0.88 = 1.5 \text{ ft}$$

(b) Compute a table of values for $W(u)$ for values of u ranging from 0.9 to 10^{-15}. Code the equation

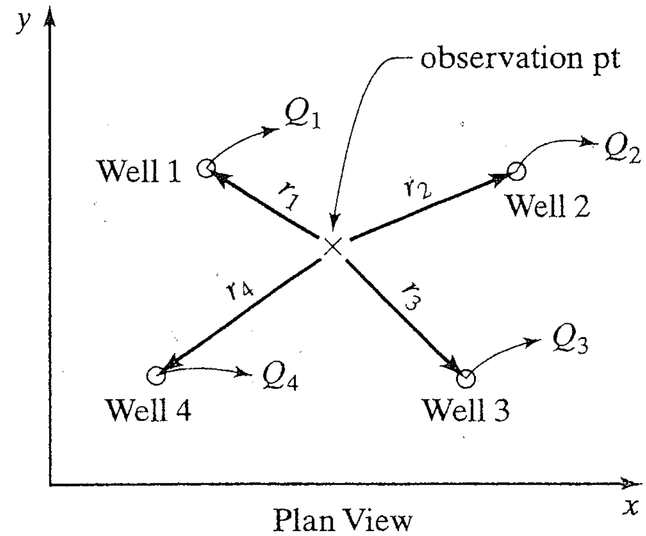
$$W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!}$$

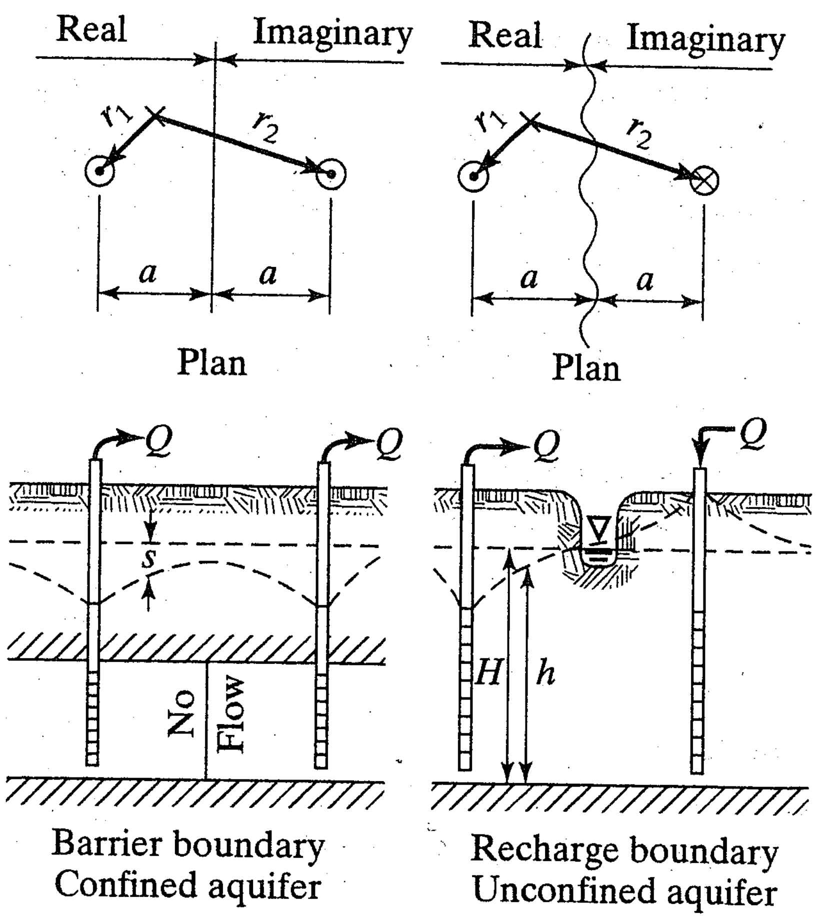
and print values of $W(u)$ in tabular form. Output is listed below.

VALUES OF W(*u*) (WELL FUNCTION OF *u*)

	u	1	2	3	4	5	6	7	8	9
X	.1E + 00	1.82	1.22	.91	.70	.56	.45	.37	.31	.26
X	.1E – 01	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
X	.1E – 02	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
X	.1E – 03	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
X	.1E – 04	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
X	.1E – 05	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
X	.1E – 06	15.54	14.85	14.44	14.15	13.93	13.75	13.59	13.46	13.34
X	.1E – 07	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
X	.1E – 08	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
X	.1E – 09	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
X	.1E – 10	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
X	.1E – 11	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
X	.1E – 12	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
X	.1E – 13	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
X	.1E – 14	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

Superposition Principle & Application for Image Wells





Superposition Principle & Application for Image Wells:

Example of Overlapping of Cones of Depression from Two Pumps in Confined Aquifer

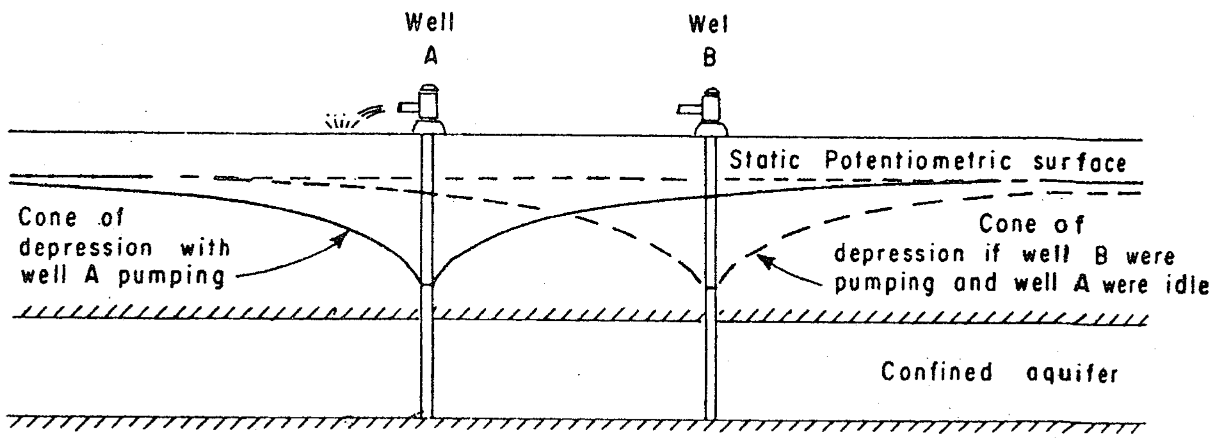


Figure 5-14. Cone of Depression When Well A or B is Pumped

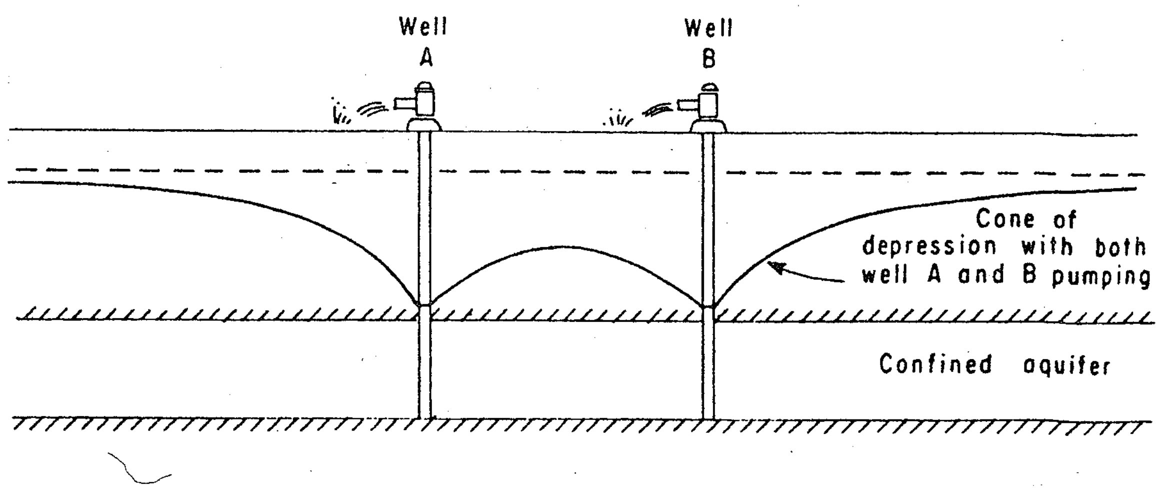
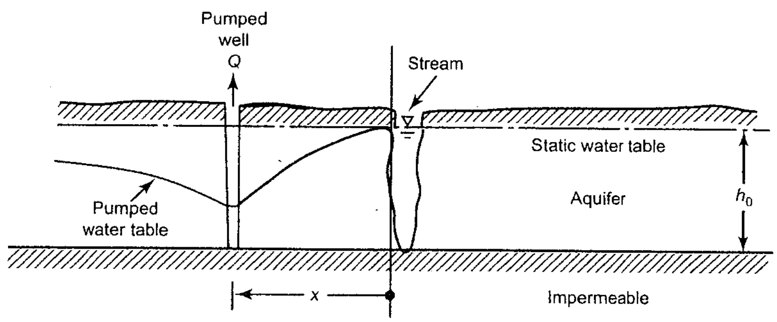


Figure 5-15. Total Drawdown Caused by Overlapping Cones of Depression

Superposition Principle & Application for Image Wells:

Well Near Stream (Stream → Image of Injecting Well)

Figure 6.21 Well near a stream and its equivalent imaginary system in an aquifer of infinite extent.



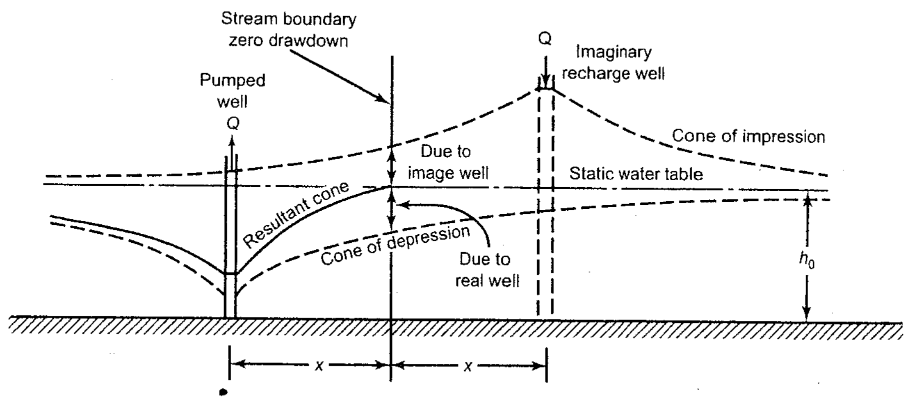
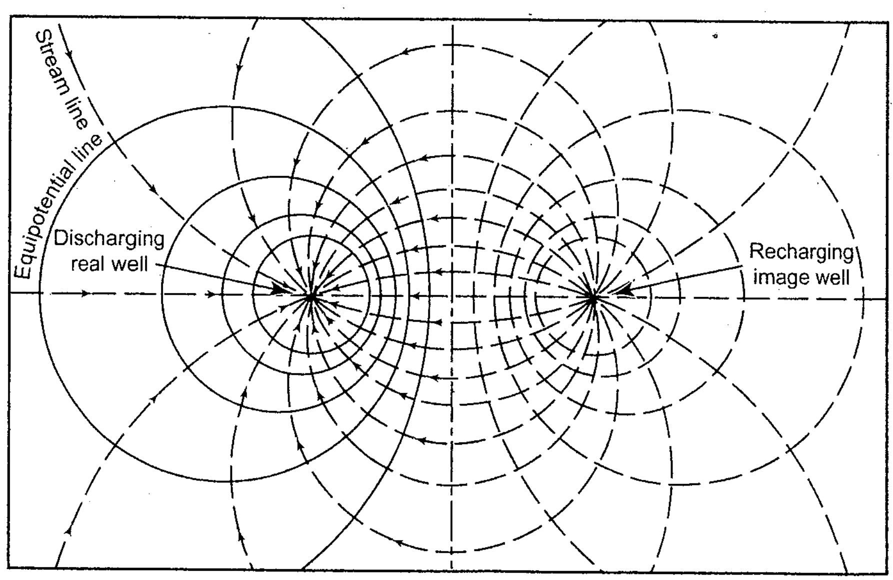


Figure 6.22 Flow net for a discharge well and its imaginary recharge well.



Superposition Principle & Application for Image Wells:

Well Near An Impermeable Barrier (Barrier → Image of Extracting Well)

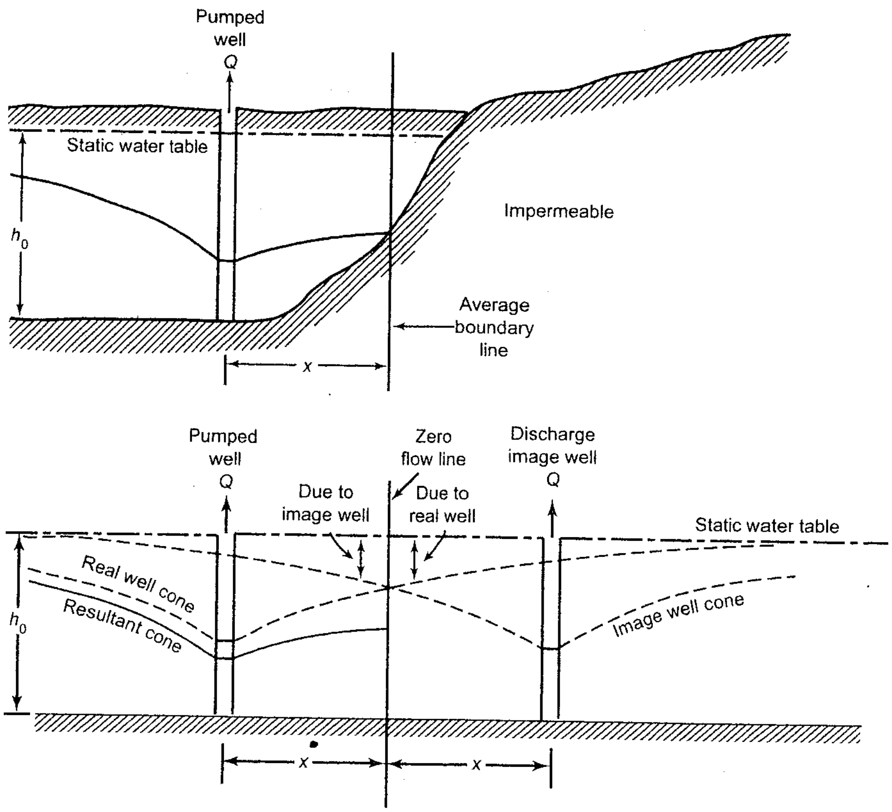
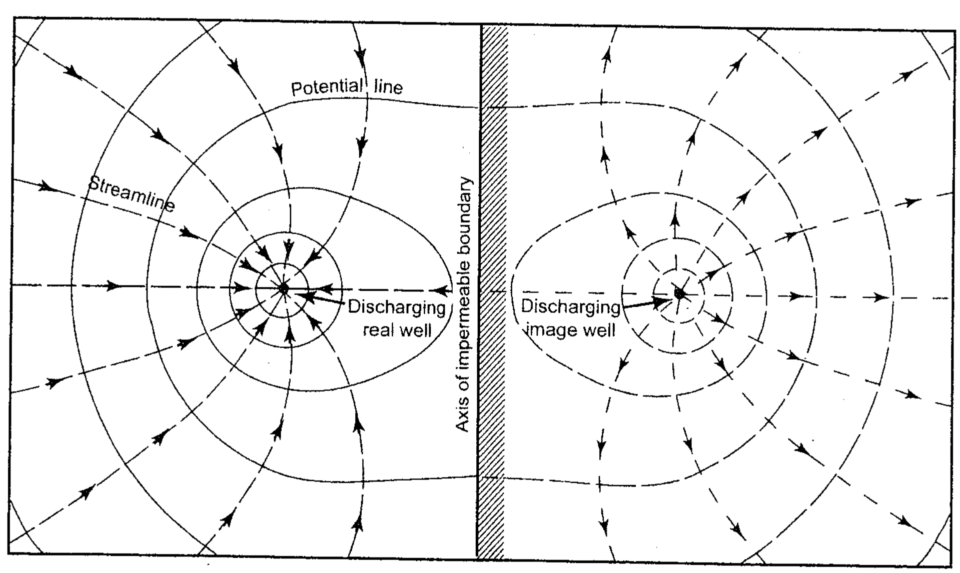


Figure 6.25 Flow net for a discharge well and its imaginary discharge well.



Superposition Principle & Application for Image Wells:

Well Near Stream (Stream → Image of Injecting Well) (Cont.)

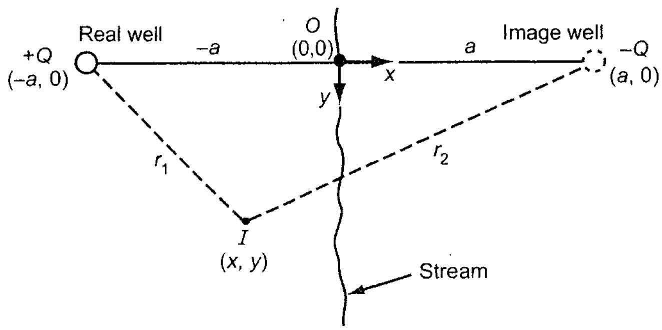


Figure 6.23 Setting up coordinates for a well near a stream with its image.

$$r_1 = \sqrt{(a-x)^2 + y^2} \qquad (6.36)$$

$$r_2 = \sqrt{(x+a)^2 + y^2} \qquad (6.37)$$

$$s = \frac{Q}{4\pi bK} \ln \frac{y^2 + (a+x)^2}{y^2 + (a-x)^2} \quad [L]$$

Superposition Principle & Application for Image Wells:

Well Near An Impermeable Barrier (Barrier → Image of Extracting Well) (Cont.)

$$s = \frac{Q}{2\pi bK} \ln \frac{R^2}{r_1 r_2} \quad [L]$$

where

r_1, r_2 are given by eqs. (6.36) and (6.37)

$R =$ radius of influence or boundary of the island