In cases where transient drawdowns are of interest, the transient drawdowns caused by the real and image wells can be superimposed, where the transient drawdown induced by each well is given by the Theis equation (Equation 15.123). Impermeable barriers are frequently associated with the rising side of a buried valley, a situation that is quite common in the northern, once-glaciated parts of the United States.

15.5.3 Other Applications

The previous examples demonstrate the fundamental reasons why the method of images works, and provide guidance for applying this method to other cases. The linearity and homogeneity of the governing differential equation guarantee that superimposed solutions will also satisfy the governing differential equation. The selection of the location(s) of image wells is controlled by the requirement that the superimposed drawdowns must meet the boundary conditions. In the case of a constant-head boundary, an image well is placed to ensure zero drawdown at the constant-head boundary; in the case of an impermeable boundary, an image well is placed to ensure that the slope of the drawdown curve is zero at the impermeable boundary.

15.6 Saltwater Intrusion

In coastal aquifers, a transition region exists where the water in the aquifer changes from freshwater to saltwater. Since saltwater is denser than freshwater, the saltwater tends to form a wedge beneath the freshwater, as shown in Figure 15.18, for the case of an unconfined aquifer. This illustration is somewhat idealized, since in reality there is not a sharp interface between freshwater and saltwater zones, but rather a "blurred" interface resulting from diffusion and mixing caused by the relative movement of the freshwater and saltwater. This blurred interface between the freshwater and saltwater zones is sometimes referred to as the *zone of salinity transition* (Prieto and Destouni, 2005) or simply the *transition zone* (Motz and Sedighi, 2009). The saltwater interface is commonly taken as the 10,000 mg/L iso-salinity line, since water with higher salinity is considered unsuitable for human use. The volume of water below the saltwater interface is called the *saltwater wedge*. Seawater within the saltwater wedge is not static, as is often assumed, but flows inland along the base of the aquifer, mixes with the seaward flowing freshwater, and also discharges to the sea. Consequently, the groundwater discharged to the sea consists of both freshwater of terrestrial origin and recycled seawater. The relative movement between the freshwater and saltwater zones is usually associated with mean groundwater flow toward the coast, tides, and temporal variations in aquifer stresses. The thickness of the freshwater zone can range from a few meters to over a 100 meters.

The intrusion of saltwater into coastal aquifers is generally of concern because of the associated deterioration in groundwater quality. Since the recommended maximum contaminant level (MCL) for chloride in drinking water is 250 mg/L and a typical chloride level in seawater is 14,000 mg/L, mixing more than 1.8% seawater with nonsaline water renders the mixture nonpotable. This percentage is even less if the freshwater contains a nonzero chloride concentration. The location of the freshwater–saltwater interface is usually tracked

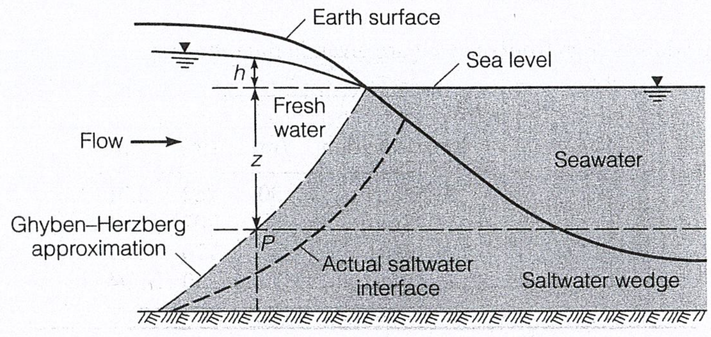


FIGURE 15.18: Saltwater interface in a coastal aquifer

in coastal aquifers using monitoring wells; however, care must be taken to use relatively short well screens to avoid within-well mixing that would bias the measured thickness of the freshwater–saltwater transition zone (Shalev et al., 2009). In the United States, saltwater intrusion has resulted in the degradation of aquifers in many coastal states and has been primarily caused by overpumping in sensitive portions of the aquifers. Some of the most seriously affected states are Florida, California, Texas, New York, and Hawaii. The impacts of saltwater intrusion in coastal aquifers are likely to increase as sea levels continue to rise in the 21st century. Reported average rates of sea-level rise are on the order of 2 mm/yr (0.08 in./yr) for the 20th century (Douglas, 1997) and on the order of 3 mm/yr (0.12 in./yr) in the years 1992–2010 (Nicholls and Cazenave, 2010). Since approximately 60% of the world's population lives within 30 km (20 mi) of the shoreline (Loaíciga et al., 2012) and groundwater is a key water-supply source in these areas, saltwater intrusion is an ever-present concern for much of the world.

An approximate method for determining the location of the saltwater interface was introduced independently by the Dutch engineer W. Badon-Ghyben* (1888) and the German engineer A. Herzberg (1901) and is called the *Ghyben–Herzberg approximation*. Under this approximation, the pressure distribution is assumed to be hydrostatic within any vertical section of the aquifer, which implicitly assumes that the streamlines are horizontal (i.e., the Dupuit assumption). Under this assumption, the hydrostatic pressure at point P in Figure 15.18 can be calculated from either the freshwater head or the saltwater head, which means that

$$\gamma_f(h + z) = \gamma_s z \tag{15.244}$$

where γ_f is the specific weight of freshwater, γ_s is the specific weight of saltwater, h is the elevation of the water table above sea level, and z is the depth of the saltwater interface below sea level. Solving Equation 15.244 for z leads to

$$z = \frac{\gamma_f}{\gamma_s - \gamma_f} h \quad \text{or} \quad z = \frac{\rho_f}{\rho_s - \rho_f} h \tag{15.245}$$

where ρ_f is the density of freshwater and ρ_s is the density of saltwater. In dealing with saltwater intrusion, both saltwater and freshwater densities are relevant, and they almost always occur in the combination $(\rho_s - \rho_f)/\rho_f$ so it is convenient to represent this combination as a single variable, ϵ, defined as

$$\epsilon = \frac{\rho_s - \rho_f}{\rho_f} \tag{15.246}$$

where ϵ is commonly called the *buoyancy factor*. Under typical conditions, $\rho_f = 1000$ kg/m^3 and $\rho_s = 1025$ kg/m^3 which yields $\epsilon = 0.025$. Combining Equations 15.245 and 15.246 gives

$$\boxed{z = \frac{\rho_f}{\rho_s - \rho_f} h = \frac{h}{\epsilon}} \tag{15.247}$$

This is called the *Ghyben–Herzberg equation*, and substituting the typical value of $\epsilon = 0.025$ into Equation 15.247 gives

$$z \approx 40\, h \tag{15.248}$$

which means that the saltwater interface will typically be found at a distance below sea level equal to 40 times the elevation of the water table above sea level. The Ghyben–Herzberg approximation also means that the slope of the saltwater interface is 40 times greater than the slope of the water table. Although the factor of 40 is commonly used, under typical densities of freshwater and saltwater, this factor can vary between 33 and 50 (Werner and Simmons, 2009). Near the shore, the static assumption used in the Ghyben–Herzberg approximation is less valid and the depth to the interface predicted by the Ghyben–Herzberg approximation

*Also known as Willem Badon Ghijben (Hendriks, 2010).

tends to be less than the actual depth observed in the field (Fitts, 2002). In fact, at the sho line the Ghyben–Herzberg approximation predicts that the saltwater interface is at sea lev while there must necessarily be a nonzero thickness of freshwater.

Assuming that the flow in the freshwater portion of the aquifer is horizontal and towa the coast, neglecting direct surface recharge (such as from rainfall), and assuming that the is no flow within the saltwater wedge, the flow rate, Q, of freshwater toward the coast can estimated using the Darcy equation

$$Q = K(h + z)\frac{dh}{dx} \qquad \text{(15.24}$$

where K is the hydraulic conductivity of the aquifer, and x is the distance inland from t shoreline. Equation 15.249 uses the Dupuit approximation, which assumes horizontal fl and equates the horizontal piezometric head gradient to the slope of the water table. Co bining Equations 15.249 and 15.247 and using the fact that $\epsilon \ll 1$ yields

$$Q = \frac{K}{\epsilon}h\frac{dh}{dx} \qquad \text{(15.2}$$

and integrating Equation 15.250 yields

$$Qx = \frac{K}{2\epsilon}h^2 + C \qquad \text{(15.2}$$

where C is an integration constant. Applying the boundary condition that $h = 0$ at $x =$ yields $C = 0$, and applying the boundary condition that $h = h_L$ at $x = L$ yields

$$\boxed{Q = \frac{K}{2L\epsilon}h_L^2} \qquad \text{(15.2}$$

This equation is particularly useful in estimating the flow of freshwater toward the coa based on the elevation, h_L, of the water table at a distance L from the coast. The water-ta profile can be estimated by combining Equations 15.251 (with $C = 0$) and 15.252 to yield

$$\boxed{h = h_L\sqrt{\frac{x}{L}}} \qquad \text{(15.2}$$

Equations 15.252 and 15.253 are derived from Equation 15.251 using the boundary co dition that $h = 0$ at $x = 0$, which requires that the saltwater interface intersect the coastl at sea level. Whereas this condition is consistent with the Ghyben–Herzberg approximati it is not physically realistic since there must be a finite width to accommodate freshwa flow as illustrated in Figure 15.19. The finite width, W, of freshwater flow at the coastline v estimated theoretically by Glover (1959) as

$$W = \frac{Q}{2\epsilon K} \qquad \text{(15.2}$$

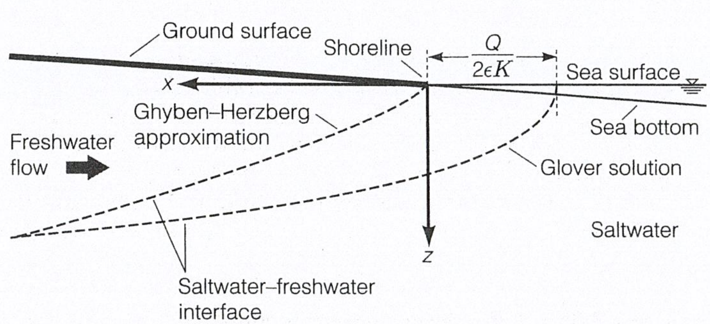


FIGURE 15.19: Saltwater interface at the shoreline

Hence, using the alternative boundary condition that $h = 0$ at $x = -W$ in Equation 15.251 yields $C = -QW$, and applying the boundary condition that $h = h_L$ at $x = L$ yields

$$\boxed{Q = \frac{K}{2(L - W)\epsilon} h_L^2} \qquad (15.255)$$

It is apparent that the difference between using the Ghyben–Herzberg approximation and the Glover approximation at the coastline is that in the latter case the distance in the x direction is replaced by $x - W$, indicating that the Glover solution shifts the saltwater interface by W toward the coastline compared to the interface estimated directly from the Ghyben–Herzberg approximation. The depth of the saltwater interface below sea level, z, at any distance, x, from the coastline can be estimated by combining Equations 15.251 and 15.247 which yields

$$\boxed{z = \sqrt{\frac{2}{K\epsilon}(Qx - C)}} \qquad (15.256)$$

where $C = 0$ corresponds to applying the Ghyben–Herzberg approximation at the coastline, and $C = -QW$ corresponds to applying the more realistic Glover (1959) solution at the coastline. All the relationships presented here neglect the influence of tidal oscillations on the location of the saltwater interface. Laboratory experiments, field measurements, and numerical models indicate that tidal influences will cause the saltwater interface to be shifted toward the coastline relative to the Glover (1959) solution (Kuan et al., 2012).

The results presented here demonstrate that a small number of piezometric head measurements can be used to obtain an estimate of the freshwater discharge of an aquifer and the location of the interface between freshwater and saltwater. In reality, groundwater flow at the coastline is not uniformly distributed over the depth of the aquifer, with most of the flow being out of the upper portion of the submerged beach (Li et al., 2008).

EXAMPLE 15.19

Measurements in a coastal aquifer indicate that the saltwater interface intersects the bottom of the aquifer approximately 2 km from the shoreline. If the hydraulic conductivity of the aquifer is 50 m/d and the bottom of the aquifer is 60 m below sea level, estimate the freshwater discharge per kilometer of shoreline.

Solution From the given data: $x = 2$ km $= 2000$ m, $K = 50$ m/d, and $z = 60$ m. Using the Glover solution, Equation 15.256 requires that

$$z = \sqrt{\frac{2}{K\epsilon}(Qx - C)} = \sqrt{\frac{2}{K\epsilon}(Qx + QW)} = \sqrt{\frac{2}{K\epsilon}\left(Qx + \frac{Q^2}{2\epsilon K}\right)} \qquad (15.257)$$

For the given information, Equation 15.257 is a quadratic equation in Q which can be expressed as

$$Q^2 + (2\epsilon Kx)Q - (K\epsilon z)^2 = 0$$

and using the quadratic formula the positive root of Q is given by

$$Q = \frac{-(2\epsilon Kx) + \sqrt{(2\epsilon Kx)^2 - 4(K\epsilon z)^2}}{2} = \epsilon K\left(\sqrt{x^2 + z^2} - x\right)$$

Assuming $\epsilon = 0.025$ and substituting the given parameters into the above equation yields

$$Q = \epsilon K\left(\sqrt{x^2 + z^2} - x\right) = (0.025)(50)\left(\sqrt{2000^2 + 60^2} - 2000\right) = 1.12\ \text{m}^2/\text{d}$$

Therefore, the freshwater discharge per kilometer of shoreline is $1.12 \times 1000 = 1120$ (m^3/d)/km.

In applying the Ghyben–Herzberg approximation, Equation 15.247, it is useful to not that the assumption of horizontal flow produces acceptable results, except near the coast line where vertical flow components become significant. In the case of confined aquifers, th Ghyben–Herzberg approximation is also applicable, with the elevation of the water tabl replaced by the elevation of the piezometric surface. Bear and Dagan (1962) have show that the length of saltwater intrusion into a horizontal confined aquifer of thickness b is pre dicted to within 5% by the Ghyben–Herzberg equation, provided that $\pi\epsilon Kb/Q > 8$, wher Q is the rate of flow of freshwater per unit breadth of the aquifer. The Ghyben–Herzber approximation generally assumes steady-state conditions and ignores the time required fo steady-state conditions to be attained. In this respect, it is generally recognized that th Ghyben–Herzberg approximation can only provide an estimate of the extent of saltwate intrusion. Numerical experiments have shown that for a steady rise in sea level the actu extent of saltwater intrusion can be either greater than or less than the estimate based o Ghyben–Herzberg approximation (Watson et al., 2010).

Besides saltwater intrusion caused by the density difference between saltwater an freshwater, a second important mechanism for saltwater intrusion is associated with th construction of unregulated coastal drainage canals. These canals allow the inland penetr tion of saltwater via tidal inflow and subsequent leakage of saltwater from the canals in the aquifer. To prevent saltwater intrusion in coastal drainage canals, salinity-control gat are typically placed at the downstream end of the canal to maintain a freshwater head (c the upstream side of the gate) over the sea elevation (on the downstream side of the gate The freshwater head should be sufficient to prevent saltwater intrusion in accordance wi the Ghyben–Herzberg equation. During periods of high runoff and when the stages in th canals are above a prescribed level, the canal gates are opened to permit drainage whi maintaining a freshwater head that is sufficient to prevent saltwater intrusion.

EXAMPLE 15.20

Consider the gated canal in a coastal aquifer illustrated in Figure 15.20. If the aquifer thickness bel the canal is 30 m, and at high tide the depth of seawater on the downstream side of the gate is 3 m, fi the minimum depth of freshwater on the upstream side of the gate that must be maintained to preve saltwater intrusion.

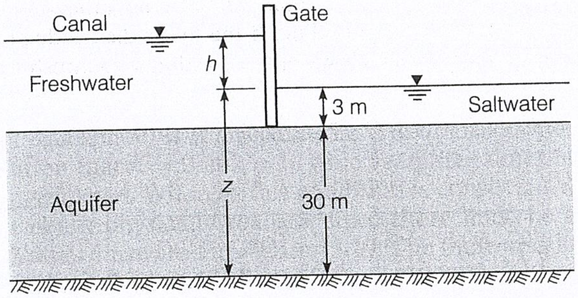


FIGURE 15.20: Gated canal

Solution The minimum elevation of the freshwater surface at the upstream side of the gate m be sufficient to maintain the saltwater interface at a depth of 33 m below sea level. According to Ghyben–Herzberg equation (Equation 15.247), the height of the freshwater surface above sea leve is given by

$$h = \epsilon z$$

where ϵ is the buoyancy factor and z is the depth of the interface below sea level. Taking $\epsilon = 0.025$ $z = 33$ m yields

$$h = (0.025)(33) = 0.83 \text{ m}$$

Therefore, the freshwater on the upstream side of the gate must be at least 0.83 m above the sea l on the downstream side of the gate. Under this condition, the total depth of freshwater in the can 3 m + 0.83 m = 3.83 m.

In addition to salinity-control gates in coastal drainage channels, other methods of c trolling saltwater intrusion include modification of pumping patterns, creation of freshw recharge areas, and installation of extraction and injection barriers. Extraction barriers are created by maintaining a continuous pumping trough with a line of wells adjacent to the sea, and injection barriers are created by injecting high-quality freshwater into a line of recharge wells to create a high-pressure ridge. In extraction barriers, seawater flows inland toward the extraction wells and freshwater flows seaward toward the extraction wells. The pumped water is brackish and is normally discharged to the sea.

Whenever water-supply wells are installed above the saltwater interface, the pumping rate from the wells must be controlled so as not to pull the saltwater up into the well. The process by which the saltwater interface rises in response to pumping is called *upconing*. This phenomenon is illustrated in Figure 15.21. Schmorak and Mercado (1969) used equations developed by Dagan and Bear (1968) to propose the following approximation of the rise height, z, of the saltwater interface in response to pumping:

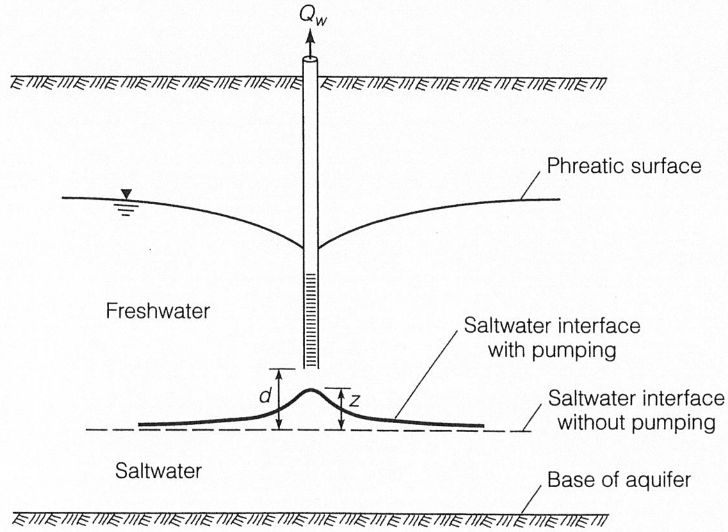
$$\boxed{z = \frac{Q_w}{2\pi d K_x \epsilon}} \tag{15.258}$$

where Q_w is the pumping rate, d is the depth of the saltwater interface below the well before pumping, and K_x is the horizontal hydraulic conductivity of the aquifer. Equation 15.258 incorporates both the Dupuit and Ghyben–Herzberg approximations, and therefore care should be taken in cases where significant deviations from these approximations occur (Nordbotten and Celia, 2006). Equation 15.258 also assumes that the thickness of the saltwater–freshwater interface is small relative to the thickness of the aquifer. In cases where these approximations are valid, experimental data indicate that Equation 15.258 provides a reasonable approximation to the height of upconing (Werner et al., 2009). Experiments have shown that whenever the rise height, z, exceeds a critical value, the saltwater interface accelerates upward toward the well. This critical rise height has been estimated to be in the range $0.3d$–$0.5d$. Taking the maximum allowable rise height to be $0.3d$ in Equation 15.258 corresponds to a pumping rate, $Q_{\max}$, given by

$$\boxed{Q_{\max} = 0.6\pi d^2 K_x \epsilon} \tag{15.259}$$

Therefore, as long as the pumping rate is less than or equal to $Q_{\max}$, pumping of freshwater above a saltwater interface remains viable, although pumping rates must remain steady to avoid blurring the interface. For anisotropic aquifers in which the vertical component of the hydraulic conductivity is less than the horizontal component, a maximum well discharge greater than that given by Equation 15.259 is possible (Chandler and McWhorter, 1975).

FIGURE 15.21: Upconing under a partially penetrating well



EXAMPLE 15.21

A well pumps at 5 L/s in a 30-m thick coastal aquifer that has a hydraulic conductivity of 100 m/d. How close can the saltwater wedge approach the well before the quality of the pumped water is affected?

Solution From the given data: $Q_w = 5$ L/s $= 432$ m^3/d, $K_x = 100$ m/d, and it can be assumed that $\epsilon = 0.025$. Equation 15.259 gives the minimum allowable distance of the saltwater wedge from the well as

$$d = \sqrt{\frac{Q_{\max}}{0.6\pi K_x \epsilon}} = \sqrt{\frac{432}{0.6\pi(100)(0.025)}} = 9.6 \text{ m}$$

Therefore, the quality of pumped water will be impacted when the saltwater interface less than or equal to 9.6 m below the pumping well.

The Ghyben–Herzberg approximation as given by Equation 15.247 can be applied to confined aquifers as illustrated in Figure 15.22(a). In this case, h represents the height of the piezometric surface above sea level, z is the depth of the saltwater interface below sea level, and these are related by

$$z = \frac{h}{\epsilon}$$

where ϵ is the buoyancy factor. In confined aquifers where the pumping well fully penetrates the aquifer, the pumping rate must be limited to ensure that the toe of the saltwater wedge does not intersect the well (Mantoglou, 2003). In fact, the toe of a saltwater wedge should not even be allowed to come near a pumping well because the toe will accelerate and be sucked into the well as it approaches the well. The case of a pumping well in a confined aquifer

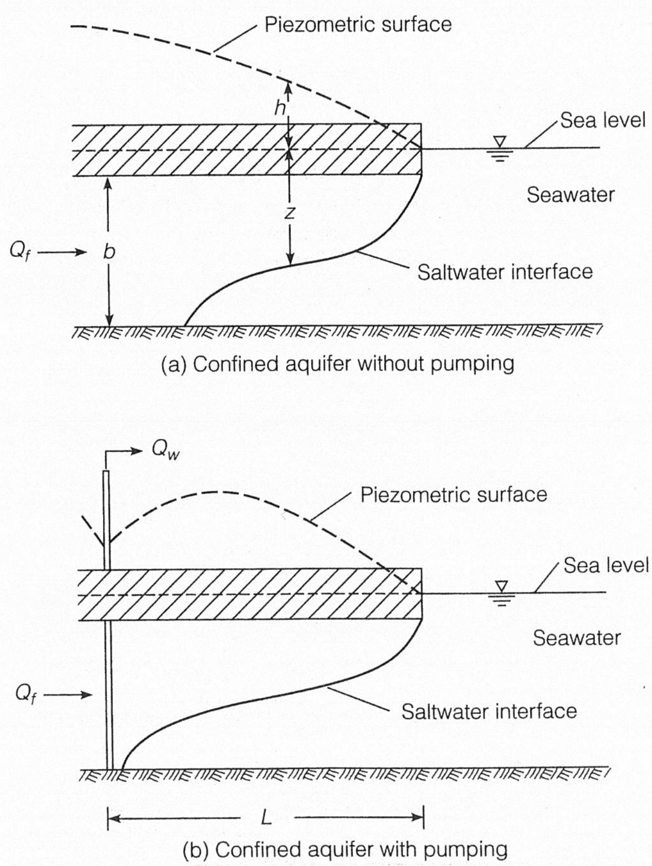


FIGURE 15.22: Saltwater intrusion in a confined aquifer

with saltwater intrusion is illustrated in Figure 15.22(b). The limitation on pumping rate was investigated by Strack (1976) who showed that the limiting pumping condition when the toe of the saltwater wedge intersects the pumping well is given by the implicit relation

$$\lambda^* = 2\left(1 - \frac{Q^*}{\pi}\right)^{\frac{1}{2}} + \frac{Q^*}{\pi}\ln\frac{\left[1 - \left(1 - \frac{Q^*}{\pi}\right)^{\frac{1}{2}}\right]}{\left[1 + \left(1 - \frac{Q^*}{\pi}\right)^{\frac{1}{2}}\right]} \tag{15.260}$$

where λ^* and Q^* are nondimensional variables defined by

$$\lambda^* = \frac{Kb\epsilon}{Lq_f} \quad \text{and} \quad Q^* = \frac{Q_{\max}}{bLq_f} \tag{15.261}$$

where K is the aquifer hydraulic conductivity (LT^{-1}), b is the aquifer thickness (L), ϵ is the buoyancy factor (dimensionless), q_f is the specific freshwater discharge from inland (LT^{-1}), L is the distance of the well from the saltwater boundary (L), and $Q_{\max}$ is the pumping rate when the toe of the saltwater interface intersects the pumping well (L^3T^{-1}). A shortcoming of Equation 15.260 is that it assumes a sharp saltwater interface and neglects mixing at the interface. As a consequence, Equation 15.260 associates the maximum pumping rate with the saltwater interface being at the well, whereas the maximum pumping rate will occur when a portion of the mixed zone intersects the pumping well. If mixing of the saltwater interface due to aquifer dispersivity is taken into account and the maximum pumping rate is associated with a salinity of 0.1% at the pumping well, then the maximum pumping rate can still be calculated using Equation 15.260; however, the buoyancy factor, ϵ, in λ^* must be replaced by a modified buoyancy factor, ϵ^*, where

$$\epsilon^* = \epsilon\left[1 - \left(\frac{\alpha_T}{b}\right)^{\frac{1}{6}}\right] \tag{15.262}$$

and α_T is the transverse dispersivity (Pool and Carrera, 2011). It is interesting to note that the modified buoyancy factor, ϵ^*, can also be used in the regular Ghyben–Herzberg equation (Equation 15.247) to estimate the location of the saltwater interface as

$$z = \frac{h}{\epsilon^*} \tag{15.263}$$

where z is the depth below sea level to the most saline portion of the mixing zone (mixing ratios between 50% and 75% seawater), and h represents either the elevation of the water table above sea level (unconfined aquifer) or the elevation of the piezometric surface above sea level (confined aquifer).

EXAMPLE 15.22

A coastal community is planning to develop a new wellfield in an area that is approximately 500 m from the coastline. A geological investigation indicates that the region is underlain by a confined aquifer of thickness 160 m, hydraulic conductivity of 80 m/d, transverse dispersivity of 1 m, and the regional freshwater flow toward the coast is approximately 0.5 m/d. It is expected that the water-supply wells will fully penetrate the aquifer. Estimate the maximum allowable pumping rate with and without taking dispersion into account.

Solution From the given data: $L = 500$ m, $b = 160$ m, $K = 80$ m/d, $\alpha_T = 1$ m, and $q_f = 0.5$ m/d. It will be assumed that $\epsilon = 0.025$.

Without Dispersion: From the given data:

$$\lambda^* = \frac{Kb\epsilon}{Lq_f} = \frac{(80)(160)(0.025)}{(500)(0.5)} = 1.28$$

Substituting λ^* into Equation 15.260 requires that

$$1.28 = 2\left(1 - \frac{Q^*}{\pi}\right)^{\frac{1}{2}} + \frac{Q^*}{\pi}\ln\frac{\left[1 - \left(1 - \frac{Q^*}{\pi}\right)^{\frac{1}{2}}\right]}{\left[1 + \left(1 - \frac{Q^*}{\pi}\right)^{\frac{1}{2}}\right]}$$

which yields $Q^* = 0.555$, and hence the maximum allowable flow rate, Q_{max}, is given by

$$Q_{\text{max}} = Q^* bLq_f = (0.555)(160)(500)(0.5) = 22,200 \text{ m}^3/\text{d} = 257 \text{ L/s}$$

With Dispersion: The modified buoyancy factor, ϵ^*, is given by Equation 15.262 as

$$\epsilon^* = \epsilon\left[1 - \left(\frac{\alpha_T}{b}\right)^{\frac{1}{6}}\right] = (0.025)\left[1 - \left(\frac{1}{160}\right)^{\frac{1}{6}}\right] = 0.0143$$

and hence λ^* is taken as

$$\lambda^* = \frac{Kb\epsilon^*}{Lq_f} = \frac{(80)(160)(0.0143)}{(500)(0.5)} = 0.732$$

Substituting λ^* into Equation 15.260 yields $Q^* = 1.24$, and hence the maximum allowable flo rate, Q_{max}, is given by

$$Q_{\text{max}} = Q^* bLq_f = (1.24)(160)(500)(0.5) = 49,600 \text{ m}^3/\text{d} = 574 \text{ L/s}$$

Taking dispersion into account increases the maximum allowable pumping rate from 257 L/s to 574 L It is apparent that neglecting dispersion of the saltwater interface can lead to very conservative es mates of the maximum allowable pumping rate.

Besides single-well cases, models have been developed to control saltwater intrusic in cases where multiple wells are used in coastal aquifers (e.g., Mantoglou and Papantonio 2008). The interconnectedness of water-supply wells in coastal aquifers allows pumping rat to be redistributed to account for seasonal and aperiodic shifts in water availability, deman and saltwater intrusion (e.g., St. Germain et al., 2008).

Classification of saline groundwater. *Saline groundwater* is a general term used to descril groundwater containing more than 1000 mg/L of total dissolved solids. There are sever classification schemes for groundwater based on total dissolved solids, and a widely cite one, initially proposed by Carroll (1962), is given in Table 15.8.

TABLE 15.8: Classification of Saline Groundwater

Classification	Total dissolved solids (mg/L)
Freshwater	0–1000
Brackish water	1000–10,000
Saline water	10,000–100,000
Brine	>100,000

Source: Carroll (1962).

Seawater has a total dissolved solids concentration of approximately 35,000 mg/L. Other forms of saline groundwater include *connate water** that was originally buried along with the aquifer material, water salinized by contact with soluble salts in the porous formation where it is situated, and water in regions with shallow water tables where evapotranspiration concentrates the salts in solution.

Problems

15.1. Consider a two-layer stratified aquifer between two reservoirs. The water surfaces in the reservoirs are at elevations 5 m and 4 m NGVD, respectively; the ground surface between the aquifers is at elevation 10 m NGVD; the top layer of the aquifer extends from ground surface down to elevation −10 m NGVD, and the base of the aquifer (and reservoirs) is at elevation −20 m NGVD. The hydraulic conductivity of the top layer of the aquifer is 50 m/d and that of the bottom layer is 100 m/d. If the reservoirs are 2 km apart, find the equation of the phreatic surface and the flow rate between the reservoirs. Neglect surface recharge.

15.2. Show that the flow rate, Q, between two reservoirs separated by a two-layer aquifer can be expressed as

$$Q = \frac{K_1}{2L}\left(h_L^2 - h_R^2\right) + (K_1 - K_2)\frac{b_2}{L}(h_R - h_L)$$

15.3. Consider the case of a fully penetrating canal shown in Figure 15.23 in which the drawdowns at a distance L from the sides of the canal are s_L and s_R on the left-hand and right-hand sides of the canal, respectively, and the effective hydraulic conductivity of the aquifer is K. Derive an expression for the leakage out of the canal per unit length of canal. Calculate the leakage when $K =$ 30 m/d, $H = 20$ m, $L = 70$ m, and $s_L = s_R = 5$ cm.

15.4. Derive the general equation for the phreatic surface in a two-layer aquifer between two reservoirs when the recharge, $N(x)$, is not equal to zero. [*Hint*: An equation similar to Equation 15.8, but with an additional term to account for recharge.]

15.5. The equation describing the phreatic surface in a two-layer aquifer between two reservoirs has been shown to be

$$\frac{K_1}{2}h^2 + (K_2 - K_1)b_2 h + \frac{Nx^2}{2} = C_1 x + C_2$$

where C_1 and C_2 are constants given by

$$C_1 = \frac{K_1}{2L}\left(h_R^2 - h_L^2\right) + (K_2 - K_1)b_2\frac{(h_R - h_L)}{L} + \frac{NL}{2}$$

and

$$C_2 = \frac{K_1}{2}h_L^2 + (K_2 - K_1)b_2 h_L$$

where N is the recharge rate between the two reservoirs. This equation describes a mounded phreatic surface, with flow to the left of the mound peak going toward the left-hand reservoir, and flow to the right of the mound peak going toward the right-hand reservoir. Derive an expression for the location of the mound peak. It has been stated that the mound peak will always be located between the two reservoirs. Use your derived expression to determine whether this statement is true or false.

15.6. Derive the general equation for the phreatic surface in a three-layer aquifer between two reservoirs. Neglect surface recharge.

15.7. A well pumps at 400 L/s from a confined aquifer whose thickness is 24 m. If the drawdown 50 m from the well is 1 m and the drawdown 100 m from the well is 0.5 m, then calculate the hydraulic conductivity and transmissivity of the aquifer. Do you expect the drawdowns at 50 m and 100 m from the well to approach a steady state? Explain your answer. If the radius of the pumping well is 0.5 m and the drawdown at the pumping well is measured as 4 m, then calculate the radial distance to where the drawdown is equal to zero. Why is the steady-state drawdown equation not valid beyond this distance?

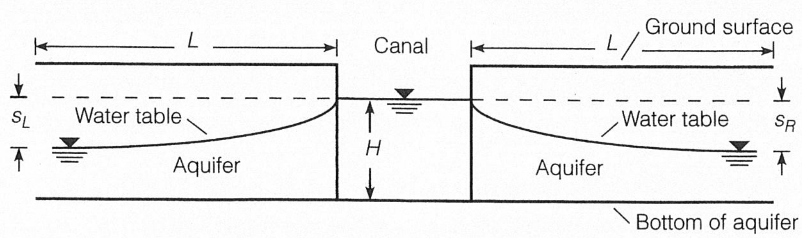


FIGURE 15.23: Leakage from a fully penetrating canal

*The word *connate* is derived from the latin word *connatus*, which means "born together."

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