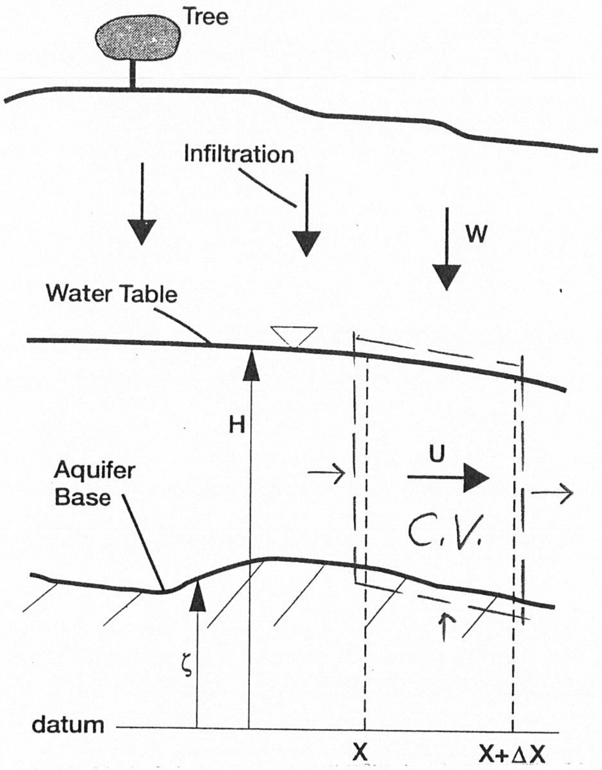

CONSERVATION OF MASS (OR CONTINUITY EQ'N) RELATIONS

FIGURE 2.3.1 Cross-section of an unconfined aquifer

Non-steady state:

$H = f(x, y, z, t)$

Steady state:

$H = g(x, y, z)$

Hydraulic Approach to grounwater flow:

a) Mostaquifers are longer (km's) than thicker (10's of meters)

b) Law of Refraction of flow lines suggest flow is mostly horizontal. Thus, on a large scale "flow in aquifers is mostly horizontal and vertical in confining beds."

∴ Velocity has 2 main components, q_x & q_y (w/ $q_z \simeq 0$)

∴ Head varies only w/ horizontal location and time or $h = f(x, y, t)$

except at partially-penetrating wells, by rivers, among others, but it meets it $\sim (1.5 - 2.0) \times$ the aquifer thickness

The Dupuit-Forchheimer Assumptions
(for an unconfined aquifer)

1) $h(x,y,z,t) \neq f(z) \therefore h(x,y,t) \rightarrow$ Mostly horizontal Q
2) Discharge Q $\propto$ slope of water table or $H(x,y,t)$
3) ρ & $n \simeq$ constant in unconfined aquifers
4) Fluxes or q_s are q_x or $U_x = -K_x (H-\xi)\frac{\partial H}{\partial x}$

q_y or $U_y = -K_y (H-\xi)\frac{\partial H}{\partial y}$

Example of Derivation of Equation (and applications)

From the continuity equation:

rate of volume accumulation or (storage) = net volume flux out − rate volume flux in ± rate volume sources or sinks

From Figure 2.3.1 between x & $x+\Delta x$, using Darcy's Law:

$$S_y \frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left[K(H-\xi)\right]\frac{\partial H}{\partial x} + \bar{W} \qquad (2.3.9)$$

2nd order Partial D.E. (or PDE)

that should be solved for Initial and Boundary conditions

a) via analytical solutions for simplified settings and
b) numerical solutions (or "matrix algebra")

Some possible simplifications are:

If flow is steady: $S_y \frac{\partial H}{\partial t} = 0$

$$\therefore K \frac{d}{dx}\left(H \frac{dH}{dx}\right) + W = 0 \quad (\text{if } \xi = 0)$$

2nd order Ordinary D.E. or ODE

If 1-D situations (Eq. 2.3.9).

$$H \frac{dH}{dx} = \frac{1}{2} \frac{dH^2}{dx}$$

$$\therefore \frac{d}{dx}\left(\frac{dH^2}{dx}\right) + 2\frac{W}{K} = 0$$

If W = constant, after integration,

$$\therefore H^2 + \frac{Wx^2}{K} = Ax + B, \quad A \,\&\, B = \text{contants of integration.}$$

If 2-D situations (Eq. 2.3.16)

$$\underbrace{\nabla^2(H^2)}_{\frac{\partial^2(H^2)}{\partial x^2} + \frac{\partial^2(H^2)}{\partial y^2} + \frac{\partial^2(H^2)}{\partial z^2}} + 2W/K = 0 \quad (2.3.17)$$

(Poisson's Eq.)

Laplacian Operator (Appendix A, p. 534).

whose solution in 1-D for radial coordinates yields

$$\frac{1}{r}\frac{d}{dr}\left(r \frac{dH^2}{dr}\right) + \frac{2W}{K} = 0$$

$$\therefore H^2 + \frac{Wr^2}{2K} = A \ln(r) + B \quad (2.3.18)$$

or Dupuit's Eq.
(for unconfined aquifers)