CWR 1204.

To solve for the flow rates in each of the pipes in Fig. 4.13, the nine equations (six linear and three nonlinear) must be solved.

Many variations of methods are available for solving the system of equations representing conservation of mass and energy in a pipe network. The Hardy Cross and linear methods are presented here. The Hardy Cross method traditionally included in most texts since the 1940's is amiable to manual computations as well as computer programs. The alternative more computationally efficient linear method is incorporated in the widely applied computer models discussed in Section 4.7. The linear method is a very stable numerical procedure that is very effective in computer modeling of pipe systems, including complex networks with thousands of pipes. Both the linear and Hardy Cross methods compute the discharge in each pipe of a network.

After the flows have been computed for all pipes in the network, the elevation of the hydraulic grade line and the pressure are computed for each junction node. The hydraulic grade line elevation for any junction node is equal to the elevation of a fixed-grade node minus the algebraic sum of headlosses in the pipes connecting the fixed-grade node and the junction node. The headloss in a pipe is considered positive if the flow in the pipe is away from the fixed-grade node and toward the junction node. The pressure (P) at the junction node (at ground elevation) is computed as

$$P = (El_{HGL} - El_{GD})\gamma_{\omega} \quad \textbf{(4.63)}$$

where El_{HGL} is the elevation of the HGL at the junction node, El_{GD} is the ground elevation at the junction node, and γ_{ω} is the unit weight of water.

4.5.1 Hardy Cross Method

Example in support of Section 4.4.

The Hardy Cross method of pipe network analyses requires an initial estimate of flow in each pipe so that the continuity equation for the junction nodes are satisfied. The loop (both closed and pseudo) equations are solved iteratively one at a time until the correction for each loop is within an acceptable magnitude. When the loop equations are solved simultaneously, it is generally referred to as the Newton–Raphson method of pipe network analysis.

If Q_i is the correct flow rate for pipe i and q_i is the assumed flow rate (or the flow rate from the previous iteration), then

$$Q_i = q_i + \Delta \quad \textbf{(4.64)}$$

where Δ is a correction term to be applied to all (N) pipes in the loop. The closed loop equation becomes

$$\sum_{i=1}^{N} K_i(q_i + \Delta)^n = 0 \quad \textbf{(4.65)}$$

Expanding

$$\sum_{i=1}^{N} K_i q_i^n + \sum_{i=1}^{N} nK_i\Delta q_i^{n-1} + \frac{n-1}{2}\sum_{i=1}^{N} nK_i\Delta^2 q_i^{n-2} + \cdots = 0 \quad \textbf{(4.66)}$$

Using only the first two terms in the binomial expansion and solving for Δ yields

$$\Delta = -\frac{\sum_{i=1}^{N} K_i q_i^n}{\sum_{i=1}^{N} |nK_i q_i^{n-1}|} \tag{4.67}$$

for the closed loops and

$$\Delta = -\frac{\sum_{i=1}^{N} K_i q_i^n - (El_B - El_A)}{\sum_{i=1}^{N} |nK_i q_i^{n-1}|} \tag{4.68}$$

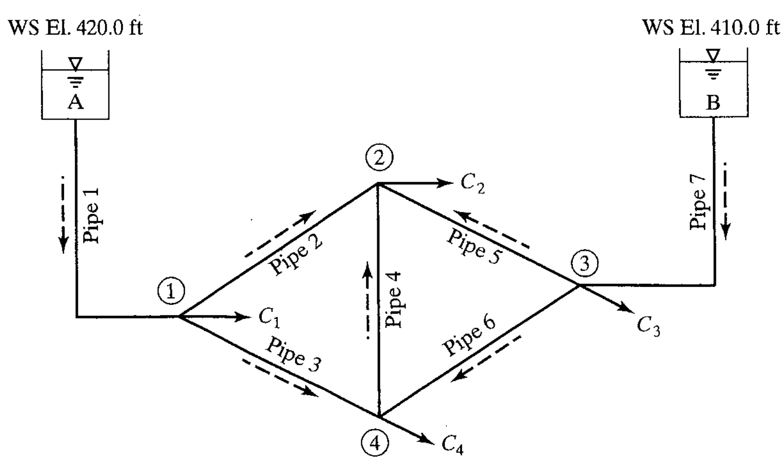
for the pseudo-loop.

The flow (q) and headloss term ($H_L = Kq^n$) for each pipe is considered positive if the flow is in the clockwise direction around the loop. Each term in the denominator can be considered as $n \times H_L/q$ and is always positive. The same correction term (Δ) is applied to all pipes in a loop. A positive correction term is added to the flow in all pipes that have flow in a clockwise direction around the loop and subtracted from the flow in all pipes that have flow in the counter clockwise direction around the loop. The continuity equations remain in balance after the flow in the pipes for a loop have been corrected.

Because only the first two terms were used in the binomial expansion of the headloss equation and because pipes that are in more than one loop have multiple corrections, the process is iterative. After applying one iterative correction to all loops, the process is repeated until convergence is achieved.

Example 4.14 Hardy Cross Pipe Network Problem

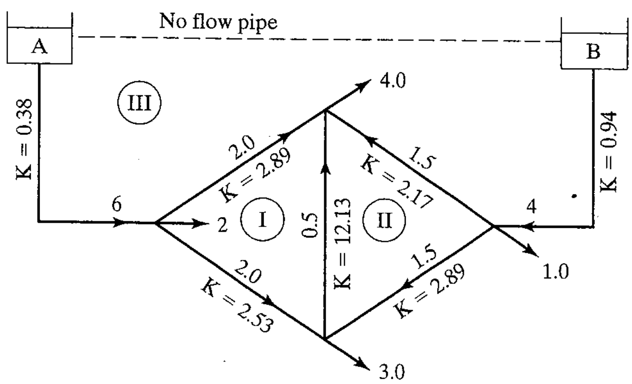
Determine the flow rate in each line and the pressure at each junction node for the pipe network in the sketch below using the Hardy Cross method of analysis. The pipe and junction data are listed below. The pipe area and headloss K values have also been computed and are listed in the table below.



Line	Nodes	Length ft	Diameter in.	f	A ft^2	K
1	A–1	1,000	12	0.015	0.78	0.38
2	1–2	800	8	0.019	0.35	2.89
3	1–4	700	8	0.019	0.35	2.53
4	4–2	750	6	0.020	0.196	12.13
5	3–2	600	8	0.019	0.35	2.17
6	3–4	800	8	0.019	0.35	2.89
7	B–3	900	10	0.017	0.55	0.94

Junction	Elevation ft	Demand cfs
1	320	2.0
2	330	4.0
3	310	1.0
4	300	3.0
	Total Demand	10.0

The Hardy Cross method requires an initial estimate of flow in each pipe such that the continuity equation is satisfied for each junction node. The estimated flows are shown below along with the K values for each pipe and the three loops.



For the two closed loops (Eq. 4.67)

$$\Delta_I = -\frac{\Sigma K q_i^n}{n\Sigma|Kq^{n-1}|} = -\frac{2.89 \times 2.0^2 - 12.13 \times 0.5^2 - 2.53 \times 2.0^2}{2(2.89 \times 2.0 + 12.13 \times 0.5 + 2.53 \times 2.0)} = +0.05$$

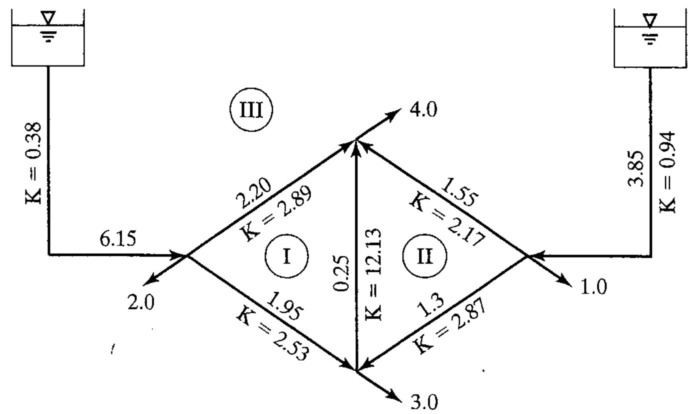
$$\Delta_{II} = -\frac{-2.17 \times 1.5^2 + 2.89 \times 1.5^2 + 12.13 \times 0.5^2}{2(2.17 \times 1.5 + 2.89 \times 1.5 + 12.13 \times 0.5)} = -0.2$$

For the pseudo-loop (Eq. 4.68)

$$\Delta_{III} = -\frac{\Sigma K q_i^n - El_B + El_A}{n\Sigma|Kq_i^{n-1}|}$$

$$= -\frac{0.94 \times 4^2 + 2.17 \times 1.5^2 - 2.89 \times 2.0^2 - 0.38 \times 6.0^2 + 10.0}{2(0.94 \times 4.0 + 2.17 \times 1.5 + 2.89 \times 2.0 + 0.38 \times 6.0)} = -0.15$$

The adjusted flows for each line are shown below. Lines 2, 4, and 5 are included in two loops and are adjusted twice. After the flows are adjusted, the continuity equation remains satisfied at each junction node.



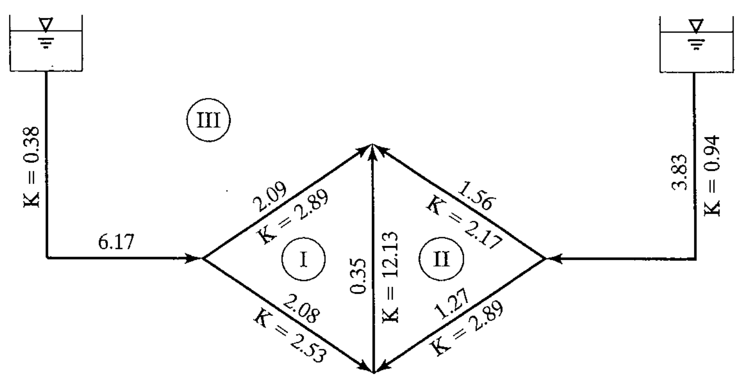
The correction terms for the second iteration are

$$\Delta_I = -\frac{2.89 \times 2.2^2 - 12.13 \times 0.25^2 - 2.53 \times 1.95^2}{2(2.89 \times 2.2 + 12.13 \times 0.25 + 2.53 \times 1.95)} = -0.13$$

$$\Delta_{II} = -\frac{-2.17 \times 1.55^2 + 2.89 \times 1.3^2 + 12.13 \times 0.25^2}{2(2.17 \times 1.55 + 2.89 \times 1.3 + 12.13 \times 0.25)} = -0.03$$

$$\Delta_{III} = -\frac{0.94 \times 3.85^2 + 2.17 \times 1.55^2 - 2.89 \times 2.20^2 - 0.38 \times 6.15^2 + 10}{2(0.94 \times 3.85 + 2.17 \times 1.55 + 2.89 \times 2.20 + 0.38 \times 6.15)} = -0.02$$

The adjusted flow rates for the second iteration are shown below.



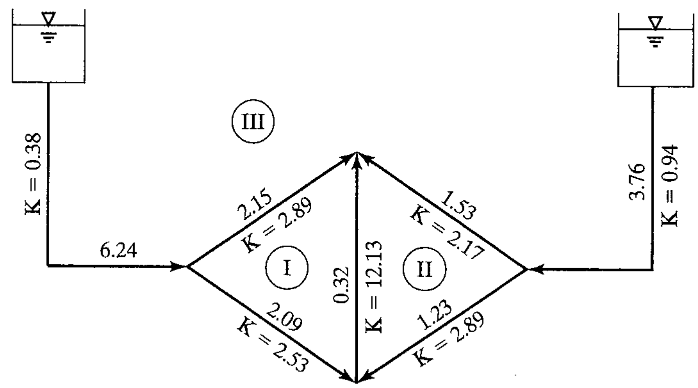
The correction terms for the third iteration are

$$\Delta_I = -\frac{2.89 \times 2.09^2 - 12.13 \times 0.35^2 - 2.53 \times 2.08^2}{2(2.89 \times 2.09 + 12.13 \times 0.35 + 2.53 \times 2.08)} = -0.01$$

$$\Delta_{II} = -\frac{-2.17 \times 1.56^2 + 2.89 \times 1.27^2 + 12.13 \times 0.35^2}{2(2.17 \times 1.56 + 2.89 \times 1.27 + 12.13 \times 0.35)} = -0.04$$

$$\Delta_{III} = -\frac{0.94 \times 3.83^2 + 2.17 \times 1.56^2 - 2.89 \times 2.09^2 - 0.38 \times 6.17^2 + 10}{2(0.94 \times 3.83 + 2.17 \times 1.56 + 2.89 \times 2.09 + 0.38 \times 6.17)} = -0.07$$

The adjusted flow rates for the third iteration are shown below



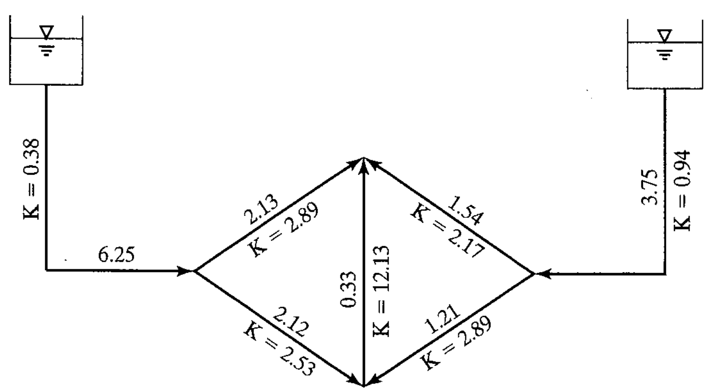
The correction terms for the fourth iteration are

$$\Delta_I = -\frac{2.89 \times 2.15^2 - 12.13 \times 0.32^2 - 2.53 \times 2.09^2}{2(2.89 \times 2.15 + 12.13 \times 0.32 + 2.53 \times 2.09)} = -0.03$$

$$\Delta_{II} = -\frac{-2.17 \times 1.53^2 + 2.89 \times 1.23^2 + 12.13 \times 0.32^2}{2(2.17 \times 1.53 + 2.89 \times 1.23 + 12.13 \times 0.32)} = -0.02$$

$$\Delta_{III} = -\frac{0.94 \times 3.76^2 + 2.17 \times 1.53^2 - 2.89 \times 2.15^2 - 0.38 \times 6.24^2 + 10}{2(0.94 \times 3.76 + 2.17 \times 1.53 + 2.89 \times 2.15 + 0.38 \times 6.24)} = -0.01$$

The adjusted flow rates for the fourth iteration are shown below.



The correction terms for the fifth iteration are

$$\Delta_I = -\frac{2.89 \times 2.13^2 - 12.13 \times 0.33^2 - 2.53 \times 2.12^2}{2(2.89 \times 2.13 + 12.13 \times 0.33 + 2.53 \times 2.12)} = -0.01$$

$$\Delta_{II} = -\frac{-2.17 \times 1.54^2 + 2.89 \times 1.21^2 + 12.13 \times 0.33^2}{2(2.17 \times 1.54 + 2.89 \times 1.21 + 12.13 \times 0.33)} = -0.02$$

$$\Delta_{III} = -\frac{0.94 \times 3.75^2 + 2.17 \times 1.54^2 - 2.89 \times 2.13^2 - 0.38 \times 6.25^2 + 10}{2(0.94 \times 3.75 + 2.17 \times 1.54 + 2.89 \times 2.13 + 0.38 \times 6.25)} = -0.01$$

The final flow rates are listed below along with the headloss for each line.

Line	Nodes	K	Q cfs	H_L ft
1	A–1	0.38	6.26	14.9
2	1–2	2.89	2.13	13.1
3	1–4	2.53	2.13	11.5
4	4–2	12.13	0.32	1.2
5	3–2	2.17	1.55	5.2
6	3–4	2.89	1.19	4.1
7	B–3	0.94	3.74	13.1

The pressure at each node is computed in the table below.

Node	Elevation ft	Line	H_L ft	HGL ft	P/γ ft	P psi
A	420			420.0		
		A–1	−14.9			
1	320			405.1	85.1	37
		1–2	−13.1			
2	330			392.0	62.0	27
		2–3	+5.2			
3	310			397.2	87.2	38
		3–4	−4.1			
4	300			393.1	93.1	40
		3–B	+13.1			
B	410			410.3		

4.5.2 Linear Method

A major difficulty with the Hardy Cross method of pipe network analysis is that the method requires an initial estimate of flow in each pipe. The initial estimates of flow must be reasonably accurate or the Hardy Cross method will not converge to a solution. With the linear method of analysis, all the equations (both continuity

Source: Wurbs & James
Prentice Hall, 2002