4.3 CULVERTS

A culvert is a cross-drainage structure used to convey water from one side of a highway, railroad, or canal to the other side. If the culvert exit and entrance are submerged, the hydraulics is the same as a pipe connecting two reservoirs. If the entrance is not submerged, then the structure is analyzed as open channel flow. The entrance is generally considered submerged if $H > 1.2D$, where H is the headwater depth and D is the pipe diameter or the height of a box culvert (Fig. 4.7). The energy equation for the culvert in Fig. 4.7 is

$$Z_{HW} - Z_{TW} = H_L \tag{4.27}$$

where Z_{HW} is the headwater surface elevation, Z_{TW} is the tailwater elevation, and H_L is the headloss. When the tailwater is below the crown (top) of the pipe at the outlet, the elevation of the crown of the pipe is used for Z_{TW} in Eq. 4.27 to compute the headloss. If there is no downstream channel and the outlet of the culvert discharges into the atmosphere, the elevation of the centerline of the culvert at the outlet is used for Z_{TW} in Eq. 4.27 to compute the headloss.

The headloss in the culvert includes both pipe friction and minor losses. Traditionally, the Mannings equation has been used in culvert hydraulics to compute friction losses. The headloss equation is

$$H_L = K_e \frac{V^2}{2g} + L \frac{n^2 V^2}{R^{4/3} C_m^2} + K_x \frac{V^2}{2g} \tag{4.28}$$

or

$$H_L = \frac{V^2}{2g}\left[K_e + K_f + K_x\right] \tag{4.29}$$

where K_e is the entrance loss coefficient (Fig. 4.8), K_f is the friction loss coefficient, K_x is the exit loss coefficient, and R is the hydraulic radius. Assuming the tailwater velocity is small relative to the velocity in the culvert, K_x is approximately 1.0. The friction loss coefficient is

$$K_f = \frac{L n^2 2g}{R^{4/3} C_m^2} \tag{4.30}$$

By substituting $V = Q/A$ in Eq. 4.29 and solving for Q gives the discharge rate for culverts flowing full (called outlet control)

$$Q = N_p A C_c \sqrt{2gH_L} \tag{4.31}$$

where

$$C_c = (K_e + K_f + 1.0)^{-1/2} \tag{4.32}$$

N_p = number of same size culverts in parallel.

If a culvert is on a steep slope, the barrel of the culvert might not flow full, and the capacity is controlled by the amount of water that can enter the entrance. For entrance-controlled culverts, the discharge is given by the orifice equation

$$Q = N_p A_e C_o \sqrt{2g(H - D/2)} \quad \textbf{(4.33)}$$

where

A_e is the area of entrance,

C_o is orifice coefficient,

H is the headwater depth (headwater elevation minus pipe invert elevation at inlet), and $H - D/2$ is the pressure head at the centroid of the area of the entrance.

Unless the entrance is flared, the area of the entrance is the same as the area of the pipe. The orifice coefficient (C_o) ranges from 0.62 for a sharp-edged entrance to 1.0 for a rounded entrance. The capacity of a culvert is the smaller of the two discharge rates computed from Eq. 4.31 for outlet control or Eq. 4.33 for entrance control.

The orifice equation is also used to compute the discharge rate through a submerged bridge (Fig. 4.9).

$$Q = A_b C_c \sqrt{2gH_L} \quad \textbf{(4.34)}$$

where A_b is the net area of the bridge opening (gross area less area of piers), and C_c is defined by Eq. 4.32. Experimental values of C_c by the Bureau of Public Roads for typical bridges range from 0.7 to 0.9. Bridges are normally not designed to operate under submerged conditions.

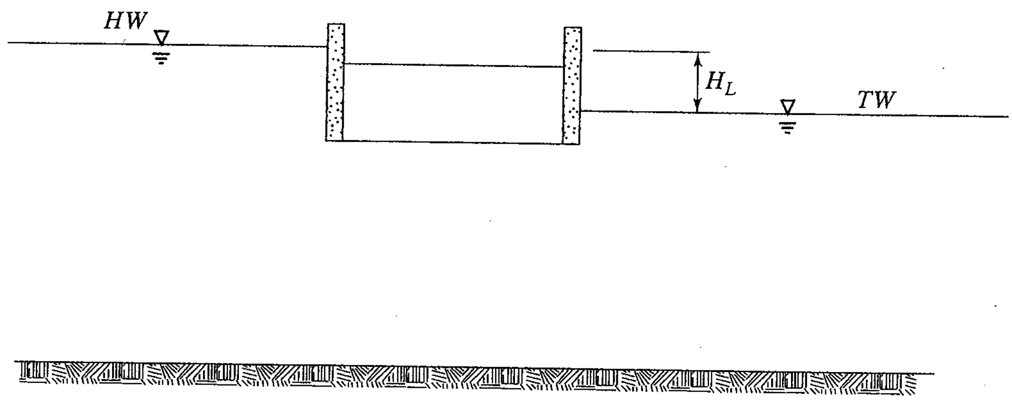
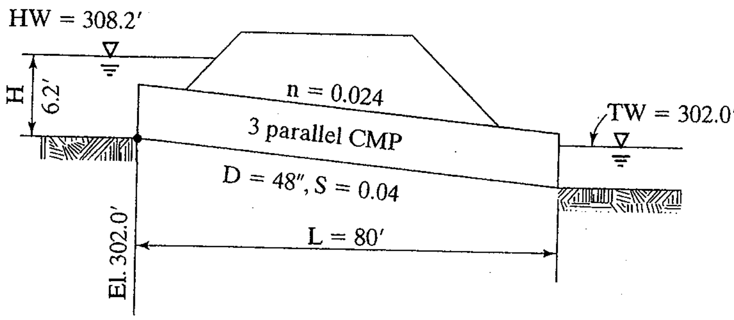


Figure 4.9 Submerged bridge.

Example 4.9 Culvert Capacity

Determine the discharge rate through three parallel corrugated metal pipe culverts shown in the sketch below. The 48-inch diameter culverts have a Manning "n" value of 0.024. The culverts are 80 ft long, laid on a slope of 4.0 percent, and have a projecting inlet and outlet. The culvert inlet invert (bottom) elevation is 302.0 ft with a headwater elevation of 308.2 ft and a tailwater elevation of 302.0 ft.



The invert elevation of the culvert at the outlet is (302.0 − 80 × 0.04) = 298.8 ft and the crown elevation at the outlet is (298.8 + 4.0) = 302.8 ft. The headloss through the culvert with outlet control is (308.2 − 302.8) = 5.4 ft. The discharge rate computed for both outlet and inlet controls are as follows:

Outlet control

$$C_c = (K_e + K_f + 1.0)^{-1/2}$$

$$K_e = 0.8 \text{ (projecting entrance Fig. 4.8)}$$

$$K_f = \frac{Ln^2 2g}{R^{4/3} C_m^2} = \frac{80 \times 0.024^2 \times 64.4}{1^{4/3}(1.49)^2} = 1.34$$

$$C_c = (0.8 + 1.34 + 1.0)^{-1/2} = 0.56$$

$$Q = N_p A C_c \sqrt{2gH_L} = 3 \times \frac{\pi 4^2}{4} \times 0.56 \sqrt{64.4 \times 5.4} = 394 \text{ cfs}$$

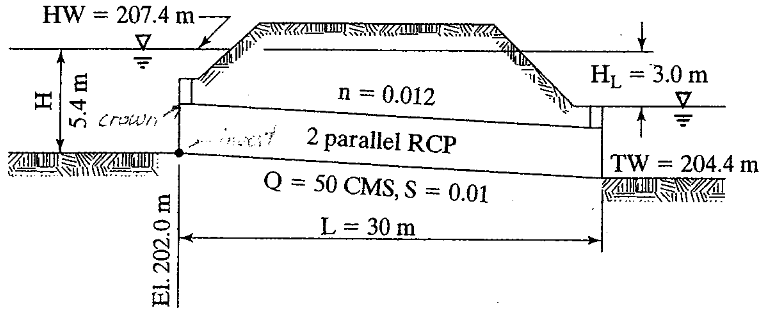
Inlet control

$$Q = N_p A C_o \sqrt{2g\left(H - \frac{D}{2}\right)} = 3 \times \frac{\pi 4^2}{4} \times 0.62 \sqrt{2g(6.2 - 2.0)} = 384 \text{ cfs}$$

Because the discharge with inlet control is less than the discharge with outlet control, the discharge through the culverts is 384 cfs.

Example 4.10 Culvert Size

Determine the size of two parallel reinforced concrete pipe culverts ($n = 0.012$) to carry a discharge of 50 cms with a headloss of 3.0 m for outlet control. As shown in the sketch below, the culverts are 30 m long with headwalls ($K_e = 0.5$) on a slope of 1.0 percent, and both exit and entrance submerged. The inlet invert elevation is 202.0 m with a headwater elevation of 207.4 m and a tailwater elevation of 204.4 m.



Equation 4.29 can be written as

$$H_L = \frac{Q^2}{A^2 2g}\left[K_e + \frac{Ln^2 2g}{R^{4/3}C_m^2} + 1.0\right]$$

where Q is the discharge per pipe, A is the area ($\pi D^2/4$), and R is the hydraulic radius ($D/4$) of the pipe. Substituting the known values into the headloss equation gives

$$3.0 = \frac{25^2}{\left(\frac{\pi D^2}{4}\right)^2 2 \times 9.81}\left[0.5 + \frac{30 \times 0.012^2 \times 19.62}{\left(\frac{D}{4}\right)^{4/3}(1.0)^2} + 1.0\right]$$

$$3.0 = \frac{51.64}{D^4}\left[1.5 + \frac{0.54}{D^{4/3}}\right]$$

or

$$D^4 - 25.82 - 9.30D^{-1.33} = 0$$

The above equation can be solved for the diameter by either trial and error or by a numerical procedure. For demonstration purposes, the Newton–Raphson numerical procedure will be used to solve for the pipe diameter.

$$F(D) = D^4 - 9.30D^{-1.33} - 25.82$$

$$F'(D) = 4D^3 + 12.37D^{-2.33}$$

The new estimate of diameter (D^+) is

$$D^+ = D - \frac{F(D)}{F'(D)}$$

where D is the old estimate of diameter. Using an initial estimate of the diameter of 2.0 m

$$F(D) = 16 - 3.7 - 25.8 = -13.5$$

$$F'(D) = 32 + 2.5 = 34.5$$

$$D^+ = 2.0 + \frac{13.5}{34.5} = 2.4 \text{ m}$$

Based on a diameter of 2.4 m

$$F(D) = 33.2 - 2.9 - 25.8 = 4.5$$

$$F'(D) = 55.3 + 1.6 = 56.9$$

$$D^+ = 2.4 - \frac{4.5}{56.9} = 2.32 \text{ m}$$

and finally based on $D = 2.32$ m

$$F(D) = 28.97 - 3.03 - 25.82 = 0.12$$

$$F'(D) = 49.9 + 1.7 = 51.6$$

$$D^+ = 2.32 - \frac{0.12}{51.6} = 2.32 \text{ m}$$

Culverts are available in sizes 7.5 ft (2.28 m) or 8 ft (2.44 m) diameter. The next culvert size is 2.44 m in diameter.

SOURCE:

Wurbs, R.A. and W. P. James
Water Resources Engineering
Prentice-Hall, 2002