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4.1.1 Continuity Equation

For steady flow in pipelines and pipe networks, water is considered incompressible, and the conservation of mass equation (continuity equation) reduces to the volumetric flow rate (Q)

$$Q = AV \tag{4.1}$$

where A is the cross-section area of the pipe, and V is the average velocity. The flow rate (Q) is measured in cu m per sec (cms) or cu ft per sec (cfs). Discharge rate (Q) may also be specified in liters per second (lps), gallons per minute (gpm), or million gallons per day (mgd). The continuity equation between cross-sections 1 and 2 of a pipe is

$$A_1V_1 = A_2V_2 \tag{4.2}$$

Junction nodes are located where two or more pipes join together. A three-pipe junction node with a constant demand (C) is shown in Fig. 4.1. The continuity equation for the junction node is

$$Q_1 - Q_2 - Q_3 - C = 0 \tag{4.3}$$

In modeling pipe networks, all demands on the system are located at junction nodes, and the flow in pipes connecting nodes is assumed to be uniform. If a major demand is located between nodes, then an additional junction node is established at the location of the demand. Equation 4.2 serves as the continuity equation for a two-pipe junction node without a demand, where the subscripts refer to the pipe number.

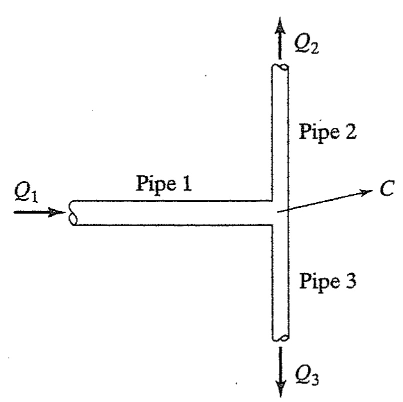
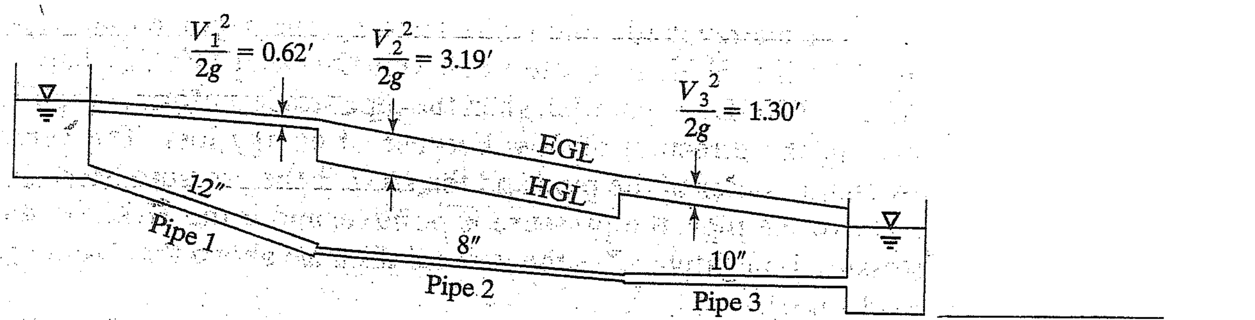


Figure 4.1 Three-pipe junction node with a constant demand.

SOURCE: Wurbs et al. (2002)

Example 4.1 Continuity Equation

If 5 cfs of water is flowing in the pipeline from the upper reservoir to the lower reservoir, determine the velocity in each line. Sketch the energy grade line (EGL) and hydraulic grade line (HGL) on the figure.



$$V_1 = \frac{Q}{A_1} = \frac{5}{0.785} = 6.34 \text{ fps}$$

$$\frac{V_1^2}{2g} = 0.62 \text{ ft}$$

$$V_2 = \frac{Q}{A_2} = \frac{5}{0.349} = 14.3 \text{ fps}$$

$$\frac{V_2^2}{2g} = 3.19 \text{ ft}$$

$$V_3 = \frac{Q}{A_3} = \frac{5}{0.545} = 9.16 \text{ fps}$$

$$\frac{V_3^2}{2g} = 1.30 \text{ ft}$$

SOURCE: Wurbs et al. (2002)

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4.1.2 Energy Equation

Figure 4.2 represents a pumped-storage hydroelectric plant where water is pumped to the upper reservoir during the off-peak power period and used to generate electricity during the peak power period. In Fig. 4.2a, water is being pumped from the lower supply reservoir through a pipeline to an upper storage reservoir. Water discharges from the upper reservoir in Fig. 4.2b through a pipeline and turbine into the lower reservoir.

The energy grade line (EGL) and hydraulic grade line (HGL) are shown in Fig. 4.2. The HGL is located one velocity head ($V^2/2g$) below the EGL. The EGL and HGL are parallel when the pipe size is uniform. The EGL slopes downward in the direction of flow because of energy loss. The vertical distance between the center of the pipe and the HGL is the pressure head (P/γ). If the HGL is above the pipe, the pressure is positive, and if the HGL is below the pipe, the pressure is negative. Z is the vertical distance above the datum (usually mean sea level—msl).

The energy equation written for flow in a pipeline (Fig. 4.3) is

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 + E_P = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + H_L \qquad \textbf{(4.4)}$$

where z is the elevation of the pipe, P/γ is the pressure head, $V^2/2g$ is the velocity head, E_P is the energy head added by the pump, and H_L is the total headloss between points 1 and 2. Each term in the energy equation has units of length and represents energy per unit weight of fluid (Newton-meters per Newton of fluid flowing or ft-lbs per lb).

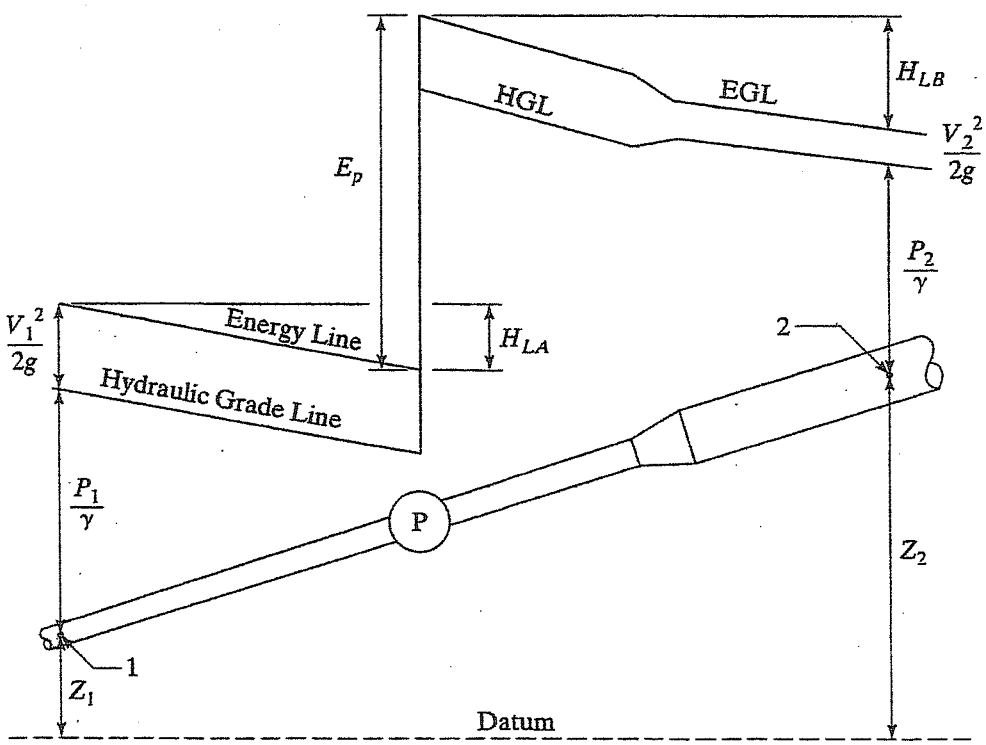

Figure 4.3 Energy equation for pipeline flow.

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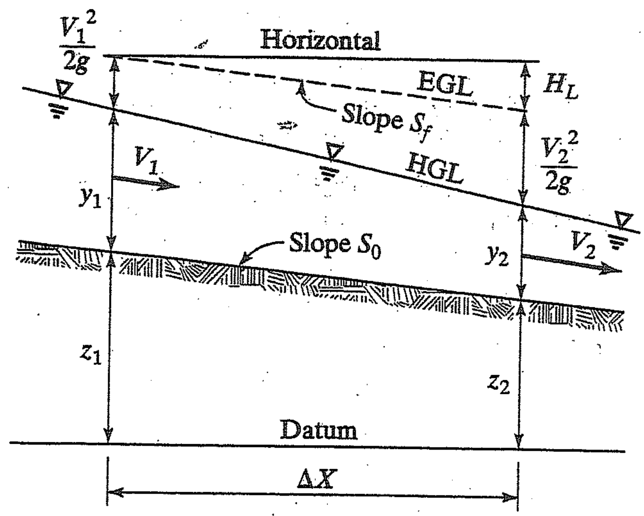
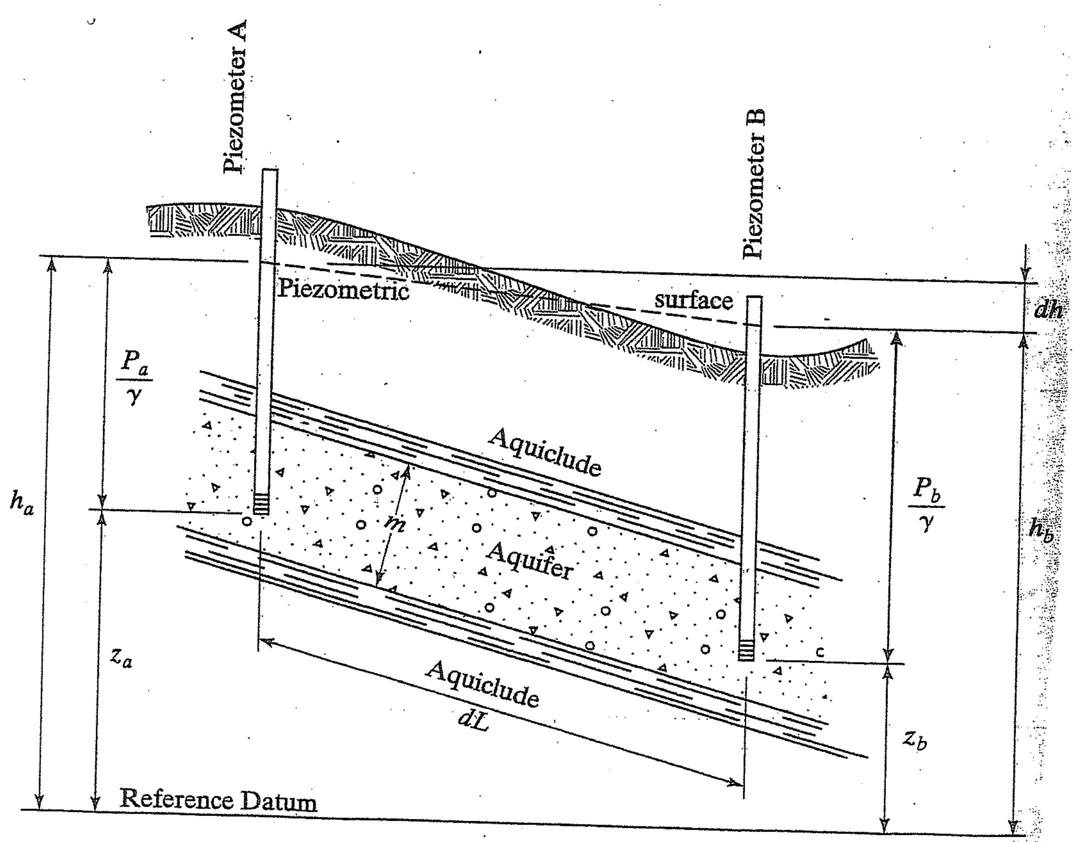


Figure 5.13 Energy equation for nonuniform flow.



Example 9.1 Groundwater flow in a confined aquifer.

SOURCE: Wurbs et al.
(2002)

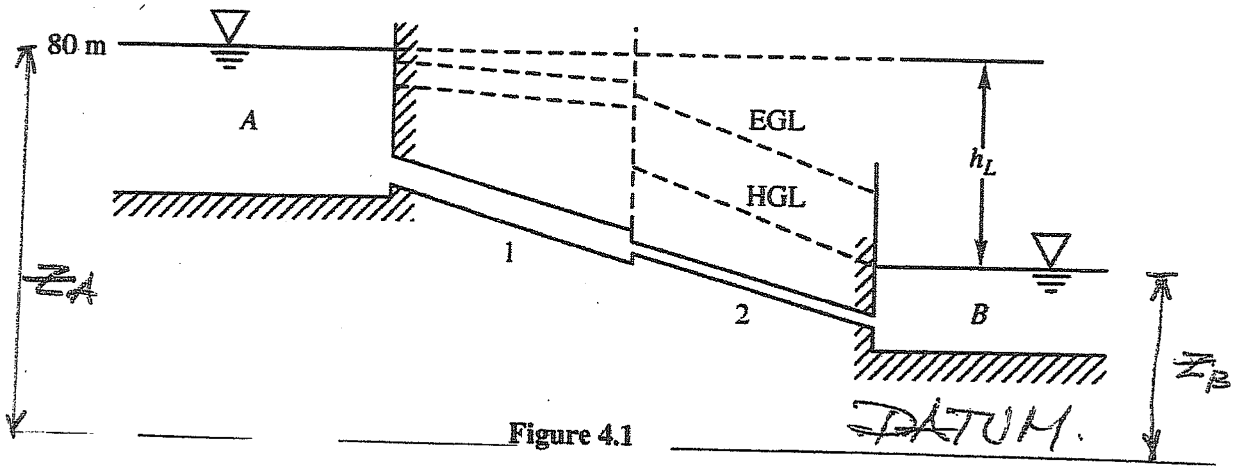


Figure 4.1

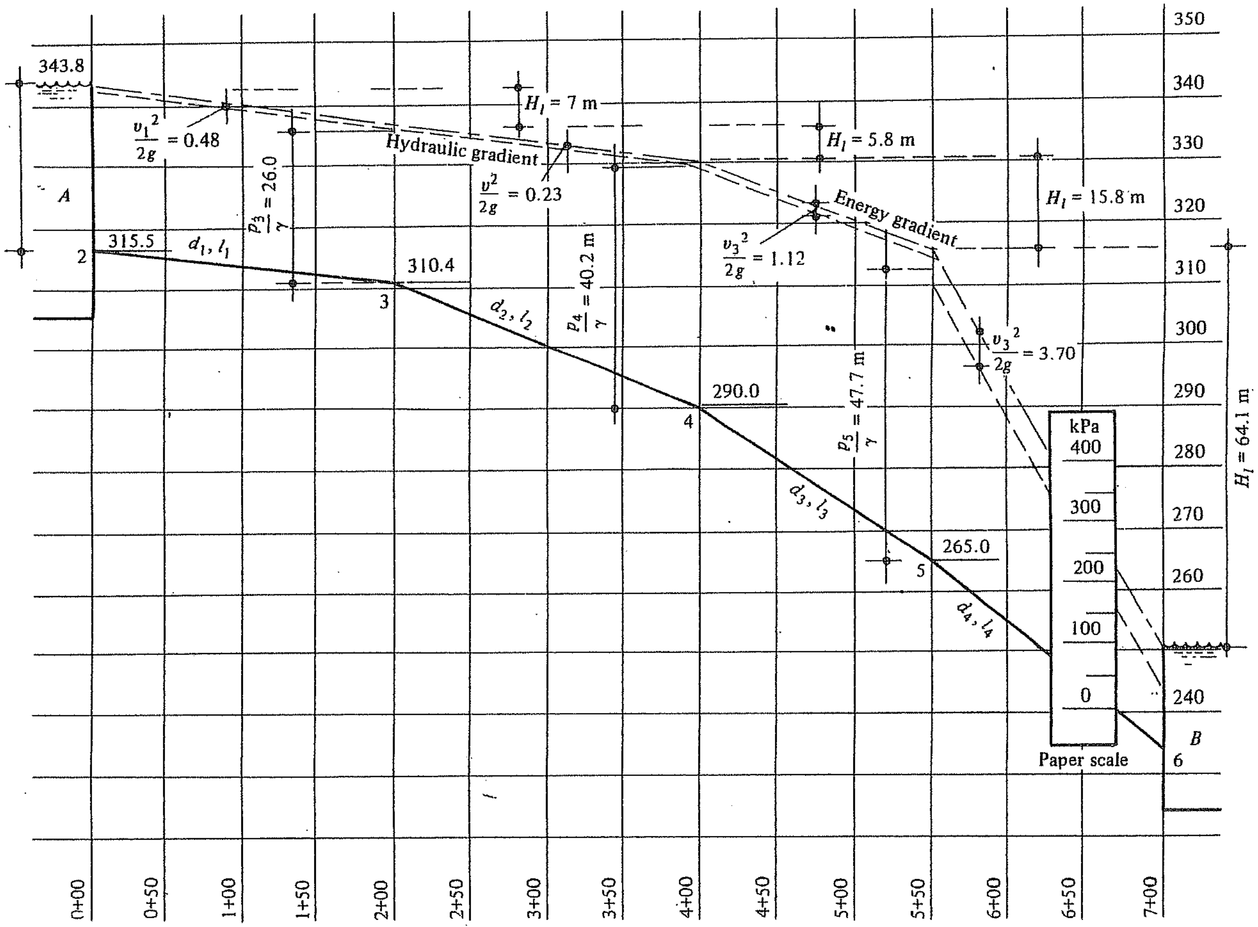


FIGURE 9.6

SOURCE: Applied Hydraulics
J. D. Kone ?
HRW, 1986

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5-7 FORCES AND STRESSES IN PIPES AND BENDS

Forces on Bends and Transitions

Bends

Because of the change in momentum that occurs with flow around a bend, the momentum equation is used to calculate the forces acting on bends and transitions. The general momentum equation for steady one-dimensional flow is

$$\sum \mathbf{F}_{\text{syst}} = \sum_{\text{c.s.}} \mathbf{V}\rho\mathbf{V} \cdot \mathbf{A} \tag{5-34}$$

Equation (5-34) is a vector form of the momentum equation using the control volume approach. Thus the subscript "syst" refers to everything inside the control volume, and the subscript "c.s." refers to the control surface. The $\Sigma\mathbf{F}$ term on the left-hand side of Eq. (5-34) includes all the external forces acting on the system (such as bend and water) within the control volume. Such forces could include forces of pressure, gravity, and the unknown force (usually acting through the pipe walls or anchor) to hold the bend or transition in place. $\mathbf{A}$ is the vector representation of area.

If Eq. (5-34) is written in scalar form and simplified for a single-stream application, such as a single stream of water flowing through a bend, we obtain

$$\sum F_x = \rho Q(V_{2x} - V_{1x}) \tag{5-34a}$$

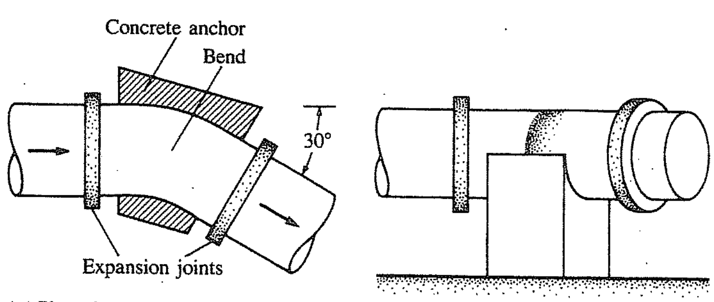
$$\sum F_y = \rho Q(V_{2y} - V_{1y}) \tag{5-34b}$$

$$\sum F_z = \rho Q(V_{2z} - V_{1z}) \tag{5-34c}$$

Example 5-10 illustrates an application of Eq. (5-34).

EXAMPLE 5-10

A 1-m diameter pipe has a 30° horizontal bend in it, as shown in Fig. A, and carries water (10°C) at a rate of 3 m^3/s. If we assume the pressure in the bend is uniform at 75 kPa gage, the volume of the bend is 1.8 m^3, and the metal in the bend weighs 4 kN, what forces must be applied to the bend by the anchor to hold the bend in place? Assume expansion joints prevent any force transmittal through the pipe walls of the pipes entering and leaving the bend.



(a) Plan view   *(b)* Elevation view

Figure A

SOLUTION

Since only a single stream of water is involved in this problem, we can use Eqs. (5-34) for the solution. Consider the control volume shown in Fig. B, and first solve for the x component of force:

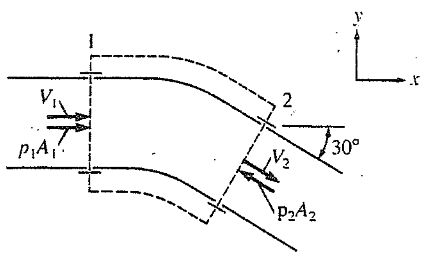


Figure B

Then

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

$$p_1A_1 - p_2A_2 \cos 30° + F_{\text{anchor},x} = 1{,}000 \cdot 3(V_{2x} - V_{1x})$$

where $p_1 = p_2 = 75{,}000$ Pa
$A_1 = A_2 = (\pi/4)D^2 = 0.785\ \text{m}^2$
$V_{2x} = (Q/A_2) \cos 30° = 3.31$ m/s
$V_{1x} = Q/A_1 = 3.82$ m/s

$$F_{\text{anchor},x} = 3{,}000(3.31 - 3.82) + 75{,}000 \times 0.785(0.866 - 1)$$
$$= -9{,}420\ \text{N}$$

Solve for F_y:

$$\sum F_y = \rho Q(V_{2y} - V_{1y})$$

$$F_{\text{anchor},y} = 1{,}000 \cdot 3(-3.82 \sin 30° - 0) - p_2A_2 \sin 30°$$
$$= -35{,}170\ \text{N}$$

Solve for F_z:

$$\sum F_z = \rho Q(V_{2z} - V_{1z})$$

$$W_{\text{bend}} + W_{\text{water}} + F_{\text{anchor},z} = 1{,}000 \cdot 3(0 - 0)$$
$$-\ 4{,}000 - 1.8 \times 9{,}810 + F_{\text{anchor},z} = 0$$

$$F_{\text{anchor},z} = +21{,}660\ \text{N}$$

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Then the total force that the anchor will have to exert on the bend will be

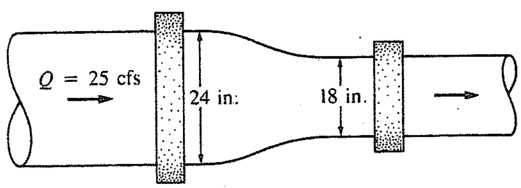
$$\mathbf{F}_{\text{anchor}} = -9{,}420\mathbf{i} - 35{,}170\mathbf{j} + 21{,}660\mathbf{k} \text{ N}$$

Transitions

The fitting between two pipes of different size is a *transition*. Because of the change in flow area and change in pressure, a longitudinal force will act on the transition. To determine the force required to hold the transition in place, the energy, momentum, and continuity equations are all applied. Example 5-11 illustrates the principles.

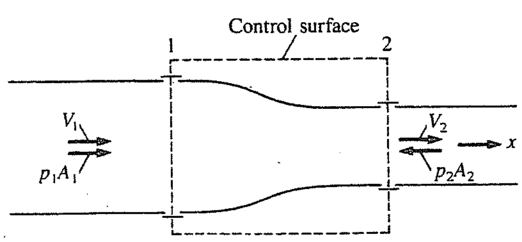
EXAMPLE 5-11

Water flows through the contraction at a rate of 25 cfs. The head loss coefficient for this particular contraction is 0.20 based on the velocity head in the smaller pipe. What longitudinal force (such as from an anchor) must be applied to the contraction to hold it in place? We assume the upstream pipe pressure is 30 psig, and expansion joints prevent force transmittal between the pipe and the contraction.



SOLUTION

Let the x direction be in the direction of flow, and let the control surface surround the transition as shown in the figure.



First solve for p_2 with the energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

where $\frac{p_1}{\gamma} = 30 \times \frac{144}{62.4} = 69.2\text{ ft}$

$$V_1 = \frac{Q}{A_1} = \frac{25}{(\pi/4)2^2} = 7.96\text{ ft/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{25}{(\pi/4) \times 1.5^2} = 14.15\text{ ft/s}$$

$$z_1 = z_2$$

$$h_L = 0.20\,\frac{V_2^2}{(2g)}$$

Then

$$\frac{p_2}{\gamma} = 69.2\text{ ft} + \frac{7.96^2}{2g} - \frac{14.15^2}{2g}(1 + 0.2)$$

$$\frac{p_2}{\gamma} = 66.45\text{ ft}$$

or

$$p_2 = 4147\text{ psf}$$

Now solve for the anchor force:

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

$$p_1A_1 - p_2A_2 + F_{\text{anchor},x} = 1.94 \cdot 25(14.15 - 7.96)$$

$$F_{\text{anchor},x} = 1.94 \cdot 25(14.15 - 7.96) + 4147 \times \left(\frac{\pi}{4}\right) \cdot 1.5^2 - 30 \cdot 144 \cdot \left(\frac{\pi}{4}\right) \cdot 2^2$$

$$= -5{,}943\text{ lb}$$

The anchor must exert a force of 5,943 lb in the negative x direction on the transition.

Note: In many cases, such as with a continuously welded steel pipe, the pipe walls are designed to carry this reaction, and no anchor block is required.

Ref.: Robertson et al. (1998).